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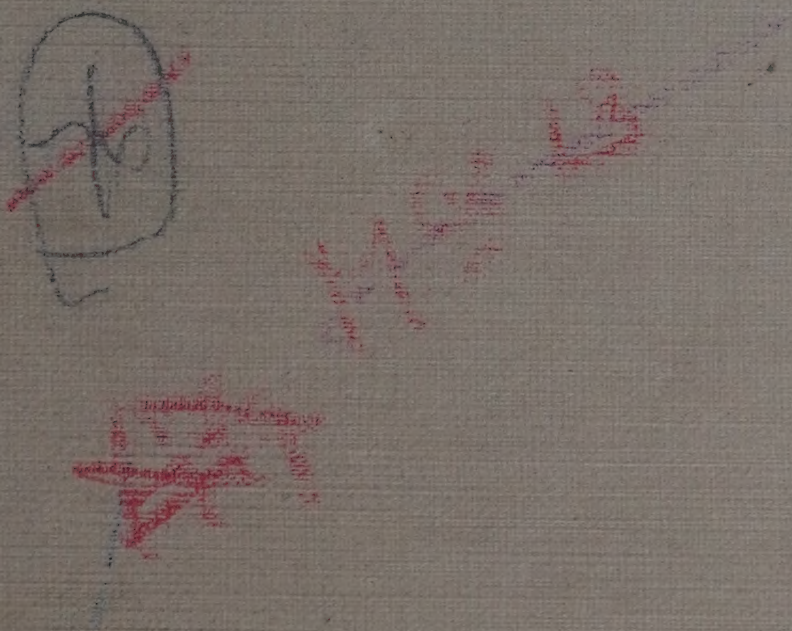

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TEST EXAMINATIONS IN MATHEMATICS

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PREFACE

THIS volume of Examination Papers is intended for use in Classes preparing for a School Certificate or Matriculation Examination.

The papers themselves are based on the modern tendencies of such Examinations taken as a whole, and while it has not been my purpose to supply the needs of the examination of any particular body, I have perhaps given more attention to those which are the most "popular," judged from the point of view of the number of candidates.

The papers are not graduated in the sense of later papers being the harder ; I have, in fact, endeavoured to make them of equal difficulty, so that, if desired, they may be taken in any order.

It is intended that seven of the eight questions in each paper shall be done in two hours, but this is clearly a matter for the discretion of the Teacher.

In Arithmetic I have been particularly careful to include questions of the type that occur, in the case of some Examining Bodies, in a combined Arithmetic and Algebra Paper, these being usually of greater difficulty than those met with in an Arithmetic Paper pure and simple. This has perhaps resulted in the Arithmetic Papers being slightly harder than is the average Arithmetic Paper. Again, I have endeavoured as far as possible to stress the present tendency

of Examiners to set questions, in an Arithmetic Paper, based essentially on algebraic or geometrical ideas. On the other hand, no attempt has been made to cope with methods peculiar to a single Authority, such as, to give one example only, that of the Northern Universities' Joint Matriculation Board in setting alternative Trigonometrical questions in the Arithmetic Paper.

In cases where I have asked for an answer to be correct to a given number of places, I have left it to the discretion of the Teacher to stipulate whether or not contracted methods shall be used.

No attempt has been made to make the wording of geometrical theorems uniform throughout, and it will be found that, in certain cases, the same theorem appears in various forms. This I regard as necessary, as Examiners are not confined to particular forms of enunciation.

Throughout I have attempted to give particular principles the prominence usually given to them by Examiners, judged by the frequency of their occurrence in official Papers.

Difficulties have, of course, arisen owing to the differences in the syllabuses of the various Authorities. For example, while the Northern Joint Board includes ratio, proportion and variation in its Algebra syllabus, London University omits them. Again, in Geometry, the former Authority includes the theorems on the properties of similar triangles, which are not required by London University. It is for this reason that I have given eight questions in each paper. Teachers preparing pupils for the London Examinations can overcome this difficulty in, say, the Geometry Papers by omitting question 8.

While many questions are original, I have not hesitated to make considerable use of questions set in recent years, and in this connection I wish to express my thanks to the following Authorities for their kind permission to use questions taken from their Papers : The Oxford and Cambridge Examination Board, The Oxford Local Examinations, The Cambridge Local Examinations, The University of London, and the University of Bristol.

I am much indebted to my colleague, Mr. L. C. Soar, B.A., for his kindly assistance in verifying the answers to many of the questions and in the reading of the proofs. Should there be any errors in the answers in spite of our care, I should be grateful for an intimation.

A. S. P.

WHITGIFT GRAMMAR SCHOOL
CROYDON
September, 1926

TEST EXAMINATIONS IN MATHEMATICS

ARITHMETIC

1a

1. The total strength of a police force is 1750. The Chief Constable wishes to distribute the rest of the force among 3 divisions of the city in proportion to their populations, which are 23,000, 69,460 and 41,630. How many police must be allocated to each division ?

2. Simplify (i) $\dfrac{(2\frac{3}{5} - 1\frac{3}{8}) \text{ of } \frac{4}{7}}{1\frac{2}{3} - \frac{1}{5} + 3\frac{1}{4} \text{ of } 1\frac{3}{5}}$,

(ii) ·257 of 38·31 × £2 12*s.* (correct to the nearest penny).

3. The area of a square field is 136,161 square feet. Find its perimeter. Given that 1 in. is equivalent to 2·54 cm., find the area in square decametres correct to the nearest tenth.

4. A trapezium has the two parallel sides of lengths 10 yards and 6 yards 2 ft., and the other two sides each of length 4 yards 1 ft. Find its area. Find also the breadth of a rectangle of the same area and of length 8 yards 1 ft.

5. A bread and butter machine cuts and butters 68 slices for every two-pound loaf. If a two-pound loaf costs 5*d.* and butter is 2*s.* 2*d.* a lb., it is found that 1000

slices of bread and butter cost £1 3*s.* 6*d.*; estimate the number of slices to 1 lb. of butter.

6. A tea merchant blends two teas costing respectively £12 and £17 a cwt., and sells the mixture at 3*s.* per lb. In what ratio must the teas be mixed if he intends to gain 20% on the cost price?

If through an accident 5% of the mixture becomes unsaleable what will he gain or lose per cent. if he still sells the remainder at 3*s.* per lb.?

7. In the year 1906 specimens of shafts up to two inches in diameter were examined for error in workmanship by the Engineering Standards Committee, the average error being given in units of 0·001 in. Twelve of these shafts were too large and had an average error of 0·1; forty-four were too small with an average error of 0·8; the remaining twelve were correct. Find the average error for the sixty-eight specimens tested. What is the average error equivalent to in inches?

8. I invest half my capital in $4\frac{1}{2}\%$ War Loan at $94\frac{1}{2}$ and the other half in New Zealand 6% stock at 108. The total income from these investments is £4173. What is the amount of capital I invested?

2a

1. Simplify

$$[13\tfrac{1}{8} - 5\tfrac{3}{7} \text{ of } 2 \times 1\tfrac{2}{19}] \div \{(1\tfrac{2}{11} - \tfrac{1}{2}) \div (5\tfrac{5}{11} - 1\tfrac{3}{4})\}.$$

Evaluate $\dfrac{3{\cdot}875 \times 0{\cdot}5461}{0{\cdot}07641}$ to three significant figures.

2. August 14th, 1926, fell on a Saturday. Find on what day February 23rd, 1929, will fall. On what day of the week will February 23rd, 1933, fall?

3. Find the compound interest on 3000 dollars in 3 years, the first year's interest being at 7 per cent. per annum, the second at 6 per cent., and the third at 5 per cent.

4. Walking at 3 miles per hour, a boy takes $2\frac{1}{4}$ minutes to walk along one side of a rectangular field, and $8\frac{1}{2}$ minutes to walk round it. Find its area in acres, and the time—to the nearest second—he would save in getting from one corner to the diagonally opposite corner by going straight across instead of along two sides.

5. Find the cost of 3 tons 13 cwt. 3 qrs. if the cost of one ton is £8 10*s*. 6*d*.

6. A bachelor's income-tax is assessed as follows :—

(i) Deduct one-sixth of income.

(ii) From what remains deduct "personal allowance" of £135. This gives "taxable income."

(iii) On the first £225 of taxable income, tax is charged at the rate of 2 shillings in the pound.

(iv) On the rest of the taxable income, tax is paid at the rate of 4 shillings in the pound.

What is the income tax paid by a bachelor whose income is £600 ?

If a bachelor pays £40 10*s*. 0*d*. income tax, what is his income ?

7. A and B exchange valves for their amplifier sets. The valve A gives to B requires a filament current of 0·6 amps. The valve B gives to A requires a filament current of 0·41 amps. How much per cent. does A save, and how much per cent. does B lose, in consumption of current ?

8. A man instructs his broker to purchase for him £300 of stock at $81\frac{1}{2}$ and £450 of stock at 110. Noticing subsequently that the first is beginning to fall and that the second has risen as high as it is likely to go he has them both sold out, the first at 79, and the second at such a price that, on the two transactions together, he makes a profit of £60. At what price was the second stock sold ? (Disregard purchase and sale charges.)

3a

1. A man spends on dress in a series of years the sums shown in the column on the right. Find the average yearly expenditure, to the nearest penny. (It is not necessary to copy out the figures given; but the sum must be given in your answer.)

	£	s.	d.
1912 ..	33	5	$2\frac{1}{2}$
1913 ..	31	15	7
1914 ..	30	14	$9\frac{1}{2}$
1915 ..	26	7	11
1916 ..	21	2	6
1917 ..	24	7	$6\frac{1}{2}$
1918 ..	31	5	9
1919 ..	13	15	8
1920 ..	34	11	10
1921 ..	24	16	1
1922 ..	22	17	5

2. (i) Simplify $\left(1\frac{1}{1}\frac{2}{3} \text{ of } \frac{3 - \frac{5}{2\,1}}{3\frac{1}{2} + 1\frac{1}{3}} - 1\right) - (2\frac{2}{2}\frac{1}{5} - \frac{1}{5}\frac{4}{})$.

(ii) Evaluate $3{\cdot}14159 \times 2{\cdot}41423 \times 12{\cdot}71845$ correct to one decimal place.

3. A gas fire burns 1 cubic foot of gas in 73 seconds. If 1000 cubic feet of gas are equivalent to 4·8 therms, and the gas costs $9\frac{1}{4}d.$ per therm, find the cost, to the nearest pound, of keeping the fire burning continuously for a year of 365 days.

4. A square lead plate 3·7 mm. thick weighs 1 kilogram 425 grams; and 1 c.c. of lead weighs 11·3 grams. Find the side of the square, in centimetres and millimetres, to the nearest millimetre.

5. Find, in acres, sq. chains, and sq. yards the area of a rectangular field 200 yards long by 132 yards wide, given that a chain is 22 yards and that 10 sq. chains make an acre. Find also the cost of ploughing the field if it costs 2*s.* 6*d.* to plough a strip 200 yards long and a yard and a half wide.

6. AOB is a diameter of a semicircle whose centre is O, and a square PQRS is such that PQ lies along AOB, and the corners R, S are on the semicircle. The radius of the semicircle is 24 feet. Calculate the length of a side of the square (join O to R).

If the figure represent a piece of ground, find the total cost (to the nearest penny) of covering the square to a depth of 6 in. with gravel costing 7*s*. 10*d*. per cubic yard, and of turfing the remainder of the semicircle at a cost of 12*s*. 8*d*. per 100 square feet. (The area of a circle is to be taken as 3·142 times the square of the radius.)

7. The sum of £3570 is invested, partly in 4 per cent. stock at 84, and partly in 5 per cent. stock at 102. The annual gross income is £172. Find how much is invested in each stock. (Neglect brokerage and stamp duty.)

8. The capital of a company is £50,000. In 1923 the gross income was £325,000, which, after payment of the working expenses, left a balance equal to 25 per cent. of the capital. In 1924 the gross income increased by 10 per cent., and the working expenses increased by 12 per cent. Find the balance, and find what per cent. it was of the capital.

4a

1. A lawn is 110 feet long and 50 feet wide, and there is no slope lengthways, but one side is 3 feet 6 inches higher than the other. How many loads (cubic yards) of soil must be shifted to make it level ? (The 50 feet may be taken as the width measured horizontally.)

2. Three men buy a business. The first pays $\frac{3}{8}$ of the total cost, the second pays four-fifteenths, and the third pays £494 10*s*. What is the price of the business ?

3. (i) Simplify, expressing the result as a proper fraction, $\dfrac{5\frac{5}{6} - 2\frac{1}{2} \text{ of } \frac{108}{175}}{(5\frac{5}{6} - 2\frac{1}{2}) \text{ of } \frac{108}{175}}$;

(ii) Find, correct to four significant figures, the value of $(63{\cdot}123 - 56{\cdot}8879) \div 0{\cdot}113017$.

4. Find the square root of 13994·89 and of ·076176.

5. A bookseller disposes of 225 copies of a certain work bound in three different styles. Of the first style he has 50 copies which he sells at 12*s*. 6*d*. each, less a discount

of 15 per cent. Of the second he has 96 copies which he sells at 8*s.* 4*d.* each, less a discount of 10 per cent. The remainder he sells at 5*s.* 7*d.* each, net. Find the total amount he obtained for the 225 books, and the average price obtained per book to the nearest penny.

6. What price expressed in cents per lb. is equivalent to 27·35 francs per kilogram when $131\frac{1}{8}$ francs = 485 cents ? (1 lb. = 0·454 kilogram.)

7. The salary of Civil Servants is made up of a fixed basic salary, and a bonus which is revised from time to time in the light of the prevalent cost of living. The variable bonus is proportional to what is known as the Index Figure of the cost of living. In the case of a certain Civil Servant, when the Index Figure is 224, his salary is £600, and when, the cost of living having decreased, the Index Figure is 172, his salary is £535. What is the basic salary, and what will be his salary when the cost of living is normal (indicated by an Index Figure of 100) ?

8. A man transfers £1000 of stock from the 4 per cents. at 95 to the $3\frac{1}{2}$ per cents. at 76. How much of the latter stock will he hold, and what will be the difference in his income ?

5a

1. A man paid the sums given below into his personal account at the bank during the year 1921. What was the largest total paid in during any one month, and the average monthly amount paid in ?

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
Jan.	52	10	0	May	88	3	2	Aug.	77	3	7
,,	37	4	10	June	15	10	0	Sep.	22	2	0
,,	12	4	11	,,	57	4	1	,,	57	4	1
,,	0	7	0	July	16	19	5	Oct.	91	18	0
Feb.	94	7	6	,,	95	19	4	,,	53	0	0
Mar.	57	4	1	,,	52	10	0	Dec.	57	4	1
				,,	0	7	0				

2. A man pays $\frac{1}{19}\frac{1}{0}$ of his income in income-tax, $\frac{1}{10}$ in rent and rates, 65% of the remainder in household and other expenses. Of the rest he pays £90 in the school fees of his children, and has left $\frac{1}{5}$ of the whole. What is his income ?

3. The productive acreage of England in 1921 is given as follows :

Arable	10,844,000
Pasture	12,698,000
Rough grazings . . .	3,253,000

Express each of these as percentages of the whole, giving your results to one decimal place.

4. Taking the area of a circle as 0·7854 times the square of the diameter, find to the nearest yard the circumference of a circular field of area 1000 square yards.

5. The price of certain food is 9*s*. 7*d*. per lb. Express this in francs per kilogram, given that £1 is equivalent to 176 francs and that 1 kilogram is equivalent to 2·2 lb. Give your answer correct to the nearest centime.

6. A housekeeper finds that by cooking with an oil stove she can save 15 cwt. of coal per month of 4 weeks. If the oil costs 1*s*. 3*d*. a gallon and she uses 5 gallons a week, what is the price of coal (to the nearest sixpence) per ton if she saves £2 6*s*. 3*d*. in half a year, i.e. 26 weeks ?

7. A man sells £2440 of 6 per cent. stock at 98 and purchases 5 per cent. stock with the proceeds. If the gain in income is £10 8*s*., what is the price of the second stock ?

8. A room is 18′ $5\frac{1}{4}$″ long, 14′ $3\frac{3}{4}$″ wide, and 13′ 6″ high ; the doors and windows occupy an area of 126 square feet. What will be the cost of papering it if the paper used costs 1*s*. 9*d*. per square yard ?

6a

1. The distance from the North Pole to the Equator measured along a meridian is ten million metres. Assuming the earth to be a sphere of radius 4000 miles, and the value of π to be 3·142, find the number of centimetres in a yard correct to the nearest millimetre.

2. At a point h feet above the surface of the sea the distance in miles to the horizon is $1{\cdot}231\sqrt{h}$. Find the greatest distance, to the nearest half-mile, at which a light 145 feet above the surface of the sea can be seen from a point on the surface of the earth.

Find, to the same degree of accuracy, the greatest distance at which the same light can be seen from the bridge of a ship 35 feet above sea-level.

3. Simplify

$$19s.\ 6\tfrac{1}{2}d. - \frac{1 - \frac{1}{2} \div \frac{1}{3} \times \frac{1}{4} + \frac{1}{5}}{(1 - \frac{1}{2}) \div \frac{1}{3} \times (\frac{1}{4} + \frac{1}{5})} \times 5s.\ 7\tfrac{1}{2}d.$$

4. A County rate of a penny in the pound for the County of London (including the City) will produce £207,855. If the amount to be raised by the rate has to be £10,158,007, what, to the nearest penny, must be the rate levied per £, and how much in excess or defect of the amount required will it produce ?

5. Find the weight of the water to the nearest ton which falls on an acre due to a rainfall of 29·4 inches. It is given that 1 cubic foot of water weighs 62·5 lb.

6. Find the difference between the simple and the compound interest on £3550 for 2 years at $4\frac{1}{2}$ per cent.

7. Find, by practice or otherwise, the value of 28 tons 15 cwt. 3 qrs. 14 lb. at £12 5*s*. per ton.

8. How much cash must be invested in a 5 per cent. stock at 88 to yield a net income of £140 after income tax at 6*s*. in the £ has been deducted ?

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7a

1. Simplify $(1 + \frac{45}{109} - \frac{26}{111})\ (3 - \frac{65}{124} + \frac{8}{115})$.

2. Find the number of minutes between 10.29 a.m. on February 13th, 1925, and 6.22 a.m. on May 11th of the same year. Express this period as a decimal of a leap year correct to the third decimal place.

3. On April 1st, 1922, there were 975,000 telephones in Great Britain, and it is estimated that the number will increase by $8\frac{1}{2}\%$ yearly. What should be the number on March 31st, 1927 (to the nearest 10,000) ?

4. Find the length of fencing required to surround a field whose area is 7776 square yards, if twice the length is three times the breadth. What would be the length of fencing if the field were a square one of the same area ?

5. A map is drawn to such a scale that a length of one unit on the map represents a length of 50,000 units on the ground. It is divided into squares, a side of each of which represents a length of half a mile. How many complete squares of this kind can be placed on a rectangular sheet of paper 16 inches by 12 inches, the sides of the paper being parallel to the sides of the squares ?

6. A farm of 5 a. 2 r. 24 p. with dwelling-house is put up for sale by auction with reserve price of £700. The house fetched £450 and the land £57 10*s.* per acre. By how much was the reserve price exceeded, and what did the auctioneer, whose commission was 5 per cent. on the proceeds of the sale, receive ?

7. A man who has £12,000 of 4 per cent. stock sells out at 75 and invests the proceeds in 6 per cent. stock at 92. Find the change in his income.

8. A man had a yearly income of £1200, out of which he had to pay an average of 11 pence in the pound income tax. At a later period, when the income tax had considerably increased, his income had risen to £1350. But his net income, after paying tax, was the

same for each year. Find the average rate of income tax to the nearest tenth of a penny in the pound at the later period.

8a

1. Find, to the nearest second, the value of 1·52931 days minus 14·287 hours.

2. Simplify $\dfrac{\frac{1}{70}-\frac{1}{99}}{1+\frac{1}{6930}} \div \dfrac{\frac{6}{7}+\frac{2}{17}-\frac{115}{119}}{\frac{5}{3}+\frac{122}{357}}$.

3. Obtain the prime factors of 364 and 598.

What is the least number which leaves the remainder 35 when divided by 598 and also when divided by 364?

4. The base of a reservoir is a rectangle 100 yards long by 50 yards wide. The two short sides are vertical, and the two long sides are inclined outwards at an angle of 45° with the vertical. Find the volume of water in gallons which will fill the reservoir to a depth of 6 feet, taking one gallon as equal to $277\frac{1}{4}$ cubic inches.

5. Of 3 articles the second is 27% dearer than the first and the first is 12% cheaper than the third. Compare the price of the second with that of the third. What is the cost of the first if the third is priced at £25 4*s*.?

6. Find the value, correct to four significant figures, of $\sqrt{(3{\cdot}4203)^2-1}$.

7. By selling £2075 of 5 per cent. stock at 97 a man is able to buy £2425 of $4\frac{1}{2}$ per cent. stock. What is the price of the second stock and what is the difference in the man's income?

8. An alloy consists of 48 parts by weight of gold to 52 of silver, the volume of the alloy being equal to the sum of the volumes of the gold and silver. If a cubic centimetre of gold weighs 19·6 grammes and of silver 10·2 grammes, what is the volume of 13·5 grammes of the alloy?

9a

1. The area of a square is the same as that of a rectangle whose length is two and a half times its breadth. Find the ratio of the perimeter of the square to that of the rectangle, giving your answer correct to the third decimal place.

2. Simplify the three fractions :—

$$\frac{\frac{3}{7}-\frac{1}{3}}{\frac{5}{9}-\frac{1}{2}+\frac{1}{7}},\quad \frac{\frac{9}{13}-\frac{5}{11}-\frac{1}{5}}{\frac{6}{11}-\frac{1}{13}-\frac{2}{5}},\quad \frac{\frac{1}{2}-\frac{5}{7}+\frac{3}{11}}{\frac{4}{11}-\frac{1}{4}},$$

and show that one of them is the geometrical mean of the other two.

3. If A does as much work in 3 days as B does in 4 days and as C does in 5 days, find how long it would take all three to do a piece of work which B and C together could have done in 20 days. Give your answer to the nearest half-day.

4. The salary of an Indian Official before the war was 2400 rupees a month ; the rupee was worth 1*s.* 4*d.*, and he used to send half his salary to England for his family. Now his salary is 4400 rupees a month, and the value of the rupee is 1*s.* 6*d.* What proportion of his salary must he send in order that his family may receive in English money half as much again as they used to ?

5. Without using the tables, find the value of $\dfrac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}-\sqrt{3}}$ to 3 significant figures.

6. The shape of a flat metal plate is an equilateral triangle surrounded by three semicircles, each described on one of the sides as diameter. The area of the plate is 58 square inches. Find the approximate length of the side of the triangle, being given that the area of a circle is $\frac{11}{14}$ of the square of its diameter.

7. Interest is paid on a sum of money half-yearly at the rate of $3\frac{3}{4}$ per cent. per annum. If the interest (simple) paid in $4\frac{1}{2}$ years is £47 5*s.*, what is the sum ?

8. An estate, after expenses of administration amounting to £291 13*s*. 4*d*. have been deducted, is divided between A and B in such a way that after a legacy duty of 5 per cent. has been deducted from A's share, and a legacy duty of 10 per cent. has been deducted from B's share, they each receive the net sum of £3562 10*s*. Find the gross value of the estate.

10a

1. The planet Neptune, discovered Sept. 23, 1846, revolves round the Sun in 60,188 of our days to the nearest day. Its discoverer died on the anniversary of that day 31 years later. Express as a decimal correct to 5 places the fraction of its orbit it had completed in that time.

2. Find the area of material required to construct an ordinary match-box of length $2\frac{1}{4}''$, the end of the box being $1\frac{1}{2}''$ by $\frac{3}{4}''$, on the assumption that the dimensions of the inside part of the box are in each case $\frac{1}{8}''$ less than those of the outside. Find also the volume of the inside part of the box, neglecting its thickness. (The outside is open at both ends.)

3. Given that 1 litre is a cubic decimetre, 1 cubic centimetre is ·061 cubic inches, and that 1 cubic foot is equivalent to 6·24 gallons, find the number of litres in 15 gallons, giving your answer correct to the nearest tenth of a litre.

4. In the course of trade three merchants X, Y, Z charter a steamer which, sailing from a port A, calls in turn at ports B, C and then returns straight back to A. The distances are AB = 800 miles, BC = 1200 miles, CA = 1500 miles. The goods carried for each man are represented in hundreds of tons in the following table:

	A to B	A to C	B to A	C to A
For X . .	2	5	0	8
For Y . .	0	6	4	2
For Z . .	5	0	0	6

If the total cost of chartering the vessel be £19,500, how much should each pay ?

5. An equilateral triangle is described in a circle of radius 20 yards. Find the length of a side, and the area, of the triangle, given that the altitude of an equilateral triangle is ·866 times the length of a side. Assuming that the ratio of the circumference of a circle to its radius is 6·283, find the ratio of the area of the triangle to that of the circle. (All answers are to be given correct to three places, lengths to be given in yards and areas in square yards.)

6. A lawn is twice as long as it is broad. Its area is 737·28 square yards. Find its length to the nearest inch.

7. By selling a house for £2373 a man gains 13% on his outlay. How much per cent. would he have gained if he had paid 100 guineas less for the house than he actually did ?

8. For a certain quarter a customer's account for electric lighting and heating showed a consumption of 437 units for light at 8*d.* a unit, and 1,151 units for heat at $2\frac{1}{2}d.$ per unit. For the corresponding quarter a year later the charge for light was reduced by a penny a unit, and that for heat by three farthings a unit, the amounts used being 384 and 946 units respectively. Also, in each case, for every complete ten units of heat consumed, one of the light units consumed was charged as a heat unit. Find the amount of the bill for each quarter.

11a

1. How many gallons of water, to the nearest hundred, can be accommodated in a water main 6 miles long and 1 foot in diameter ? You are given that 1 cubic foot of water is 6·24 gallons, and that $\pi = 3{\cdot}14$.

2. How many people can be paid £1 10*s.* 6*d.* each out

of £81 9*s.* 3*d.* ? If the remainder were spent in buying penny-halfpenny stamps, how many could be bought ?

3. For what sum should a vessel valued at £10,800 be insured in order that in the event of loss the value of the vessel and the premium may both be recovered, the premium being $5\frac{1}{2}$ per cent. of the sum insured ?

4. Simplify $\dfrac{1}{58 - \frac{47}{69}}$ and $\dfrac{1}{58 + \frac{47}{69}}$, expressing each as a decimal as far as they agree.

5. An earthwork embankment 300 feet long and 15 feet high, with vertical ends, is 24 feet wide at the bottom and 4 feet wide at the top, the sides being of a uniform slope. Find the volume of the embankment in cubic yards.

Find also the number of truck-loads of 5 tons each required to make the embankment, reckoning 100 lb. to a cubic foot.

6. A man invests £3640 partly in $3\frac{1}{2}$ per cent. stock at 56 and partly in $4\frac{1}{2}$ per cent. stock at 84. His resulting income is £205 per annum. Find how much income he would have gained or lost by investing the whole in (i) the $3\frac{1}{2}$ per cent. stock, (ii) the $4\frac{1}{2}$ per cent. stock.

7. A cricket ground measures, from boundary to boundary, 140 yards in breadth and 196 yards in length. The pitch (22 yards long) is situated exactly in the centre of the ground, the length of the pitch being parallel to the longer side of the ground. A batsman drives a ball (just as it is touching the ground) between cover-point and mid-off to the corner of the ground facing him on his right. If the ball pitches at a point 1 foot to the batsman's right of the line joining the middle stumps and $20\frac{1}{2}$ yards down the pitch from the bowler's end, find the distance it travels in getting to the boundary, to the nearest foot.

8. The population of the United States of America is to that of the British Isles (excluding the Free State) as

53 : 22. In the former 2 out of every 21 of the inhabitants have the telephone; in the latter 1 out of every 45 have the telephone.

If the two populations are taken together as one whole, how many per thousand have the telephone?

12a

1. Calculate the number of bricks required for building the walls of a rectangular barn 40 feet long by 20 feet wide externally, neglecting the space taken up by the mortar between the bricks. The walls are 15 feet high and 6 inches thick, each brick is 6 inches by 3 inches by 2 inches; and a gap 12 feet wide by 10 feet high is left for a door in the centre of one of the walls.

2. Simplify $\dfrac{3\frac{1}{4} - \frac{2}{3} \text{ of } (\frac{5}{6} - \frac{1}{11})}{\frac{2}{9} + 1\frac{7}{12} + \frac{8}{33}}$

leaving the answer as a vulgar fraction. What is the answer if the bracket is left out?

3. (i) In Canada 900 units of electricity are consumed per head as compared with 200 per head in England. How many persons would consume 711,000 units in Canada, and how many would consume the same amount in England? Assuming that one unit of electricity can be produced by 1·8 lb. of coal, what weight of coal to the nearest cwt. would be required for the above number of units?

(ii) When it is noon at Madras what time is it at Bristol, the longitudes of the two places being 80° 12′ E. and 2° 35′ W. of Greenwich respectively?

4. If there are 4047 square metres to an acre (4840 square yards), find the number of centimetres to an inch, correct to the second decimal place.

5. A rectangular field is measured by a tape whose length is supposed to be 66 feet, but which has actually shrunk 6 inches. According to this tape the field is 220 yards long and 55 yards wide. Find to the nearest

square yard how much the area of the field is over-estimated.

6. A man places £1560 in a bank at 3% per annum compound interest. At the end of a year he withdraws £844 10*s*. How much stands to his credit at the end of the second year ?

7. A puts £8000 and B puts £5000 into a business which B manages. Out of the profits at the end of the year B took £400 as salary ; each partner drew 5 per cent. on his invested capital, and the surplus was then divided between A and B in the ratio 2 : 3. What were the total profits for the year, if B's total receipts exceeded A's by £430 ; and what was A's total percentage return on his capital ?

8. A ball is dropped from a height of 100 feet. After each impact with the ground, it rises a distance which is four-fifths of the height from which it has last dropped. Find the average height to which it rises for the first four impacts. Calculate also the *total* distance through which it has moved up to the point when it makes its fifth impact.

13a

1. If the distance of the surface of the sun from the earth is 92,400,000 miles, and the velocity of light is 300,000 kilometres per second, and 1 mile is 1·61 kilometres, find the time that it takes for light to travel from the sun to the earth in minutes and seconds.

2. Find, correct to four significant figures, the value of

$$\frac{3067 \times 1{\cdot}25 + \frac{1}{2}(4132 - 3067) \times 2{\cdot}5 \times \frac{2}{3}}{\frac{1}{2}(4132 + 3067) \times 2{\cdot}5}$$

3. Evaluate $\sqrt{\frac{34567}{89721}}$ correct to three significant figures.

4. A tradesman fixes the selling price of goods at 30

per cent. above the price he paid for them. He sells half the goods at this price, a quarter at a discount of 15 per cent. on the original selling price, and the remaining quarter at a discount of 30 per cent. on the original selling price. What is his actual profit per cent. on the whole ?

5. The total rates to be paid on a certain house from October 1st, 1918, to March 31st, 1919 (both dates inclusive) amounted to £10 5*s.* 10*d.* The house was sold and the new occupier had to pay his share of the rates, for which he became liable from and including January 20th, 1919. How much did he pay to the nearest penny ?

6. A sports ground has a circular track of the same width all round. The area of the ground including the track is 12 acres, and excluding the track, 10 acres. Find the width of the track, assuming π to be $3\frac{1}{7}$.

7. A person invested £3500 in 3 per cent. stock at 105 and £2389 10*s.* in $2\frac{1}{2}$ per cent. stock at 59. If he had to pay income tax at the rate of 6*s.* 6*d.* in the £, express his net income as a percentage of the total sum invested, correct to the nearest $\frac{1}{8}$ per cent.

8. An estate of £4125, after being charged with estate duty at 3 per cent., is divided between two legatees so that the net amount one receives after paying 5 per cent. legacy duty is twice as much as the other receives after paying 10 per cent. legacy duty. What are the net amounts that the two legatees receive ?

14a

1. Find the difference between $\left(\frac{51}{15} - \frac{15}{51}\right) \div \frac{22}{7} + \frac{17}{22}$ and $\frac{51}{15} - \frac{15}{51} \div \frac{22}{7} - \frac{17}{22}$.

2. Find, to the nearest penny, the cost of 17 tons 3 cwt. 40 lb. at 78*s.* 6*d.* per ton.

3. A circular iron disc in which are punched 12 equal circular rivet holes weighs 30 lb. It is 1 inch thick and its diameter is 1 foot. Given that a cubic foot of iron weighs 480 lb., find the diameter of each of the holes, correct to the nearest thousandth of an inch. (Take $\pi = 3{\cdot}14$.)

4. One-sixth of a person's income is free from tax. In the case of a bachelor an amount £135 is then subtracted before the tax is assessed. When these two allowances have been made, the first £225 of the rest is taxed at 2 shillings in the £ and the remainder at 4 shillings in the £. What is the income of a bachelor who pays £90 10*s*. in tax?

5. A man wishes to let his house, which has cost him £2125, at a rent which gives him a net return of 4% per annum after setting aside £10 a year for repairs and paying income tax at 5 shillings in the £ on five-sixths of the gross rent. Find what rent he will require.

6. ABCD is the floor and PQRS is the sloping roof of a lean-to shed, P being vertically above A, Q above B and so on. The front is ADSP and the roof slopes backward only. (That is, there is no slope from SR to PQ.) Given that BC, PQ, PA and CR are equal respectively to 9, 10, 8 and 14 feet, find (i) AB, (ii) the total area of the four walls.

7. It is required to cover with linoleum the floor of a room in France, measuring 13·7 metres by 8·9 metres. The linoleum is purchased in England and the price, including all expenses, is 7*s*. 3*d*. per square yard. Find the total cost in francs, recollecting that only an exact number of square yards of linoleum can be sold. (Take 1 yard as ·9144 of a metre and £1 as 178 francs.)

8. One stock A pays $6\frac{1}{4}$%, and its price is 115; another stock B pays $4\frac{3}{4}$% and its price is 85. Which stock yields the greater income per £100 invested, and by how much, to the nearest penny?

Find also to the nearest pound how much a man should invest in A for each £100 he invests in B in order to get the same income from each stock.

15a

1. Simplify $\dfrac{3 \times 1\frac{1}{29} - 1\frac{71}{93} \div (1 - \frac{74}{279})}{\frac{1}{6} - \frac{1}{14} + \frac{1}{406}}$

2. Find the equivalent price, in £ *s.* *d.* per ton, of 3 francs 30 centimes per kilogram, reckoning 45 francs to the £, and $2\frac{1}{5}$ lb. to the kilogram.

3. A cask contains spirit and water in the proportion of 3 of spirit to 5 of water; a second cask contains 5 of spirit to 3 of water. Six gallons are taken from the first cask and nine gallons from the second cask, and put into a third cask, which already contains three gallons of water. Find the proportion of spirit to water in the third cask.

4. Find the square root of 1382·3524.

Is it possible by inspection to say whether or not 342·766 and 451·23 have exact square roots?

5. A sum of money lent at simple interest amounts, with the interest, to £220 10*s.* at the end of 3 years, and to £236 5*s.* at the end of 5 years. Find the original sum lent and the rate per cent. per annum.

6. The cross-section of a uniform beam of wood is a square of side 11 inches. The length of the beam is 4 yards, and through it, from end to end, is bored a circular hole of diameter 4 inches, the cylindrical hole being at right angles to the ends of the beam. What is the volume of wood left, correct to the nearest cubic inch? What percentage of the whole volume was taken away by the boring? (Take $\pi = 3{\cdot}14$.)

7. A train covers the following distances at the speeds given, and has stops at stations as indicated below; calculate the average speed throughout the journey, that

is, the speed at which it would have to travel without a stop in order to do the journey in the same time.

57 miles at 45 miles an hour ; $2\frac{1}{2}$ minutes stop.

31 miles at 38 miles an hour ; 1 minute stop.

87 miles at 47 miles an hour ; 3 minutes stop.

The remaining 106 miles of the journey at 58 miles an hour.

Give your answer in miles per hour correct to the nearest tenth.

8. A man sells £5000 of 5% War Stock, and buys with the proceeds $3\frac{1}{2}$% India stock at 56 (including expenses), thereby increasing his income by £9 7*s.* 6*d.* Find at what price he sold the War Stock.

ALGEBRA

1b

1. (i) Simplify—

$$\left\{\frac{x}{2-x}-\frac{x-3}{3x^2-7x+2}\right\}\times\left\{\frac{1}{x-1}+\frac{2}{x+1}\right\}.$$

(ii) Change the subject of the formula in $y=\dfrac{3x-7}{2x+1}$.

Verify your result when $x=3$.

2. (*a*) Find the factors of $x^2-24x-81$ and $15x^2-4x-35$.

(*b*) Solve the equation—

$$\frac{2(x+1)(4x+1)}{10x^2-x-2}=1+\frac{}{2x-1}-\frac{2}{5x+2}.$$

3. State and prove the formula for the sum of n terms of a Geometrical Progression whose first term is a, and whose common ratio is r. Find the sum of the first 12 terms of the series

$$1+\tfrac{2}{3}+\tfrac{4}{9}+\tfrac{8}{27}+\ldots$$

using logarithms to evaluate your answer.

What would be the sum of the remaining terms if the series were extended to an infinite number of terms ?

4. Solve the equations—

(i) $\frac{1}{4}(2x-9)-\frac{3}{5}(2x+\frac{1}{2})+\frac{1}{3}(4x+1)=0$;

(ii) $ax+\dfrac{b}{y}=c=bx-\dfrac{a}{y}$;

(iii) $x+2y=7,\ 3x^2+2xy+5=0.$

5. The ages of two men are in the ratio 4 : 5. In 10

years' time they will be in the ratio 21 : 25. What was the ratio of their ages 10 years ago ?

6. The following table gives the depth, y feet, of a river at distance x feet measured perpendicularly from one bank to the other.

x ..	0	10	20	30	40	50	60
y ..	4	10	18	24	24	10	0

Assuming that the bed of the river slopes uniformly between the points where the depths are taken, draw a graph to show the shape of the section of the river ; measure downwards and take a side of a small square to represent 1 foot for both x and y.

Prove that the area of the section is 880 square feet.

7. Show that, if the roots of the quadratic equation $ax^2 + bx + c = 0$ are α, β, (i) $\alpha + \beta = -\frac{b}{a}$, ii) $\alpha\beta = \frac{c}{a}$.

Form the equation whose roots are $3 + \sqrt{5}$, $3 - \sqrt{5}$.

8. If $a : b :: c : d$, prove that $\frac{a}{b} = \frac{pa + qc}{pb + qd}$.

If $\frac{x}{y} = \frac{2\sqrt{3} + 3\sqrt{5}}{4\sqrt{3} - \sqrt{5}}$ find the value of $\sqrt{6}$ in terms of x and y.

2b

1. Divide $21x^5 + 11x^4 + 112x^2 - 18x + 45$ by $7x^2 - x + 3$.

Find the values of a and b if $9x^2 - 24xy + 16y^2 - 24x + 2by + a^2$ is the square of $3x - 4y + a$.

2. Find the factors of :—

(i) $x^2 + 14x - 51$; (ii) $6x^2 - 53x + 105$;

(iii) $(ax + 1)(ax - 2) - 28$.

What are the possible values of a in order that these three expressions may have a common factor ?

3. A square plot of ground is surrounded by a gravel path 5 feet wide, the outside boundary of the gravel also

forming a square. It is desired to double the width of the path, and it is found that $1\frac{1}{4}$ times as much gravel is required for the extension as for the original path. Find the length of the side of the plot of ground.

4. Solve the equations :—

(i) $\frac{1}{10}(x-3) - \frac{1}{4}(x+7) = \frac{1}{4}\left(\frac{x}{5} + \frac{3}{4}\right) - \frac{1}{8}(3x-4)$;

(ii) $\left(\frac{x+1}{2} - 1\right)^2 = \left(\frac{x-1}{4} + 1\right)^2$;

(iii) $3x^2 - 5x - 7 = 0$.

Any roots which are not exact integers or fractions must be worked out to two decimal places.

5. Two men start work at the rate of £200 a year, salary is to be paid quarterly, each receiving £50 at the end of the first quarter. One is to receive an increase of £1 per quarter and the other £15 per year. Find the amount received by each in 12 years. Which is the better bargain, and by how much, in n years ?

6. Prove that $\log_{10}\frac{x}{y} = \log_{10}x - \log_{10}y$.

Given that $\log_{10}2 = 0{\cdot}30103$, show that $\log_{10}(2\frac{1}{2})$ is equal to 0·39794 without using log tables.

Using the tables, find the values of (i) $10^{\overline{2}\cdot 3\,4\,5}$, (ii) $\frac{(5{\cdot}67)^2}{\sqrt{0{\cdot}0765}}$.

7. Plot the graph of $y = (x-1)(x-2)(x-3)$ between $x = 0$ and $x = 4$.

Find the value of x where the above graph meets $2y = x$. Of what equation is this value of x a solution ?

8. If $a + \sqrt{b} = c + \sqrt{d}$, where a, c are rational and $\sqrt{b}$, $\sqrt{d}$ are surds, show that $a = c$, $b = d$.

Solve the equation

$$\sqrt{2(x+1)} - \sqrt{x-3} = \sqrt{3x-17}.$$

Three quantities p, q and r are such that p varies as r, and q varies inversely as the square of r; find how p is related to q.

If p is equal to unity when q is equal to 25, find the value of q when p is equal to 5.

3b

1. The two expressions $4x^2 - 4x - 3$, $4x^3 + 10x^2 - 14x - 9$ have a common factor. Find it.

ind also the values of x correct to three decimal places which make the other factor of the latter expression equal to zero.

2. (i) Simplify $\dfrac{a}{b(x-a)} - \dfrac{b}{a(x-b)}$, and find the value of the expression when $x = \dfrac{ab}{a+b}$.

(ii) If $x = 3t - 4$ and $y = 12 - 2t + 4t^2$, find, in its simplest form, the equation connecting x and y but not containing t.

3. Solve the equations—

(i) $\frac{1}{2}(x+4) - \frac{1}{3}(2x-1) = \frac{1}{12}(3x-2) + \frac{1}{4}$;

(ii) $3x - 4y = 7$, $8x + 5y = 20$;

(iii) $2x^2 + 3y^2 = 7xy$, $x + y = 12$.

4. Find the sum of n terms of a Geometrical Progression whose first term is a and whose common ratio is r. Using your formula, verify that the sum of $(n+1)$ terms exceeds the sum of n terms by the $(n+1)$th term.

A man invests £100 at 5% at the beginning of each of 6 consecutive years. How much has he at the end of the sixth year to the nearest pound ?

5. The price of petrol is reduced by a per cent., and a man uses a per cent. more; show that he saves money, and find how much per cent. he saves. If his petrol bill is reduced from £50 to £48, what is a ?

6. A set of points on the path of a projectile is given by the following data, where x is the horizontal distance

in yards of a point from the gun G, and y is the vertical distance in feet of that point above or below G, plus if above G and minus if below.

x.								
0	1000	2000	3000	4000	5000	6000	7000	8000
y.								
0	+1210	+2120	+2690	+2890	+2700	+2070	+980	−600

Plot a graph of the path from the given data, taking for x 1000 yards to the inch, and for y 1000 feet to the inch, and find (i) at what distance from G the projectile will strike the horizontal plane through G; and (ii) if the ground were horizontal for 5000 yards from G and then rose uniformly 1 foot vertically in 5 feet horizontally, find at what height above G the projectile would strike this slope.

7. (i) Find, by logarithms, the value of

$$\frac{(4{\cdot}169)^2 \times \sqrt[3]{{\cdot}6942}}{{\cdot}06859 \times (6{\cdot}3)^3};$$

(ii) Evaluate, without using the tables,

$$\left(\frac{1}{27}\right)^{-\frac{4}{9}} \div 6\tfrac{1}{4} \times \frac{1}{\sqrt[3]{4}}.$$

8. (i) A quantity x varies as the sum of two quantities y and z, and y varies inversely as z. When z is equal to 3, x is equal to $10\frac{2}{3}$, and when z is equal to -1, x is zero. Find the values of y and z when x is equal to 6.

(ii) Find the value of $\dfrac{2-\sqrt{3}}{7+4\sqrt{3}}$, giving your answer correct to one decimal place.

4b

1. If $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{2}{r}$, express (1) r in terms of u and v, (2) $r-u$ and $r-v$ in terms of u and v.

Show that $\dfrac{1}{r-u}+\dfrac{1}{r-v}=\dfrac{1}{u}+\dfrac{1}{v}$.

2. Resolve into factors $3x^2(x-1) - 4(x^2-1)$.
Solve the equation $3x^2(x-1) = 4(x^2-1)$.
Simplify—

$$\left[\frac{x^2-y^2}{x^2+y^2} \times \frac{\frac{x}{y}+\frac{y}{x}+2}{\left(\frac{x}{y}-1\right)\left(\frac{y}{x}+1\right)}\right] - \frac{2}{\frac{x}{y}+\frac{y}{x}}.$$

3. Solve the equations :—

(i) $\frac{x}{2} + 2y = 5,\ 2x - \frac{y}{7} = -8\frac{1}{2}$;

(ii) $(2x-3)(x+1)(x-3) = (2x-1)(x-1)(x-2)$;

(iii) $\frac{3}{x} - 4{\cdot}5x = 3{\cdot}6$.

4. Give a definition of $\log_{10}x$.
Given that $\log_{10}4 = {\cdot}602$; obtain, without using the tables, the values of $\log_{10}{\cdot}2$ and of $\log_{10}62{\cdot}5$, showing your method.
Use the tables to find the value of $\frac{(3{\cdot}49)^2 \times {\cdot}0631}{59{\cdot}5}$.

5. Assume the number of first-class passengers to be 4 per cent. of the number of third-class passengers for all trains and all distances, and the first-class fare to be double the third-class fare. It has been suggested that if the first-class fare were reduced to $1\frac{1}{3}$ of the third-class fare, it would pay the railways. If the receipts of the companies and the total number of passengers remained unaltered, what must be the new proportion of first- to third-class passengers ?

6. If a and l are the first and last terms of an Arithmetical Progression of n terms, prove that the sum of the n terms is $\frac{n}{2}(a+l)$.

A boy starts in an office with a salary of £120 a year and it is arranged that his salary shall increase by £15 a

year until the total sum he has received amounts to £5250, after which his salary will remain unchanged. What will his salary be (i) during the 17th year, (ii) during the 27th year ?

7. Draw the graphs of the equations $x^2 + y = 1$ and $y^2 + 2x = 3$ from $x = -2$ to $x = +2$; and thence find approximate solutions of the equations treated as simultaneous.

8. (i) Given that the ratio of $3x - 2y$ to $4x + 5y$ is equal to 5 : 7, find the ratio of $2x - y$ to $3x + y$.

(ii) A quantity x varies inversely as a quantity y, and y is the sum of a constant and a quantity proportional to the square root of another quantity z.

Given that, when z is equal to 9, x is equal to $\frac{1}{2\frac{1}{3}}$, and that, when z is equal to 8, x is equal to $\frac{5\sqrt{2} - 7}{2}$, find the relation between x and z. From this relation find z in terms of x.

5b

1. From the formula $x = \frac{1}{2}\sqrt{(y + 1)(y - 3)}$ obtain

(1) the values of x when $y = 6{\cdot}2$;

(2) the values of y when $x = \frac{3}{4}$.

Obtain also the corresponding formula which gives y in terms of x.

2. Find the factors of (1) $8x^2 + 14x - 15$;

(2) $1 + (a - b^2)c^2 - ab^2c^4$.

The first of the above expressions and the expression $8x^3 - 10x^2 + 23x - 15$ have a common factor. Find it.

3. A train running between two towns arrives at its destination 10 minutes late when it runs at 48 miles an hour and 16 minutes late when it runs at 45 miles an hour. Find the distance between the two towns and the schedule time for the journey.

4. (1) Simplify $\dfrac{a-b}{(a+x)(b+x)} + \dfrac{b-c}{(b+x)(c+x)}$;

(2) Solve the equation $x^2 - 2{\cdot}4x - 0{\cdot}63 = 0$, giving your answers correct to 2 places of decimals.

5. (1) What meanings do you assign to $x^{-\frac{1}{2}}$, $x^{\frac{1}{2}}$, x^0. (No proofs are required.)

(2) Prove, from elementary principles, that $x^p \times x^q = x^{p+q}$, where p and q are positive integers.

(3) Find the value of $\sqrt{\dfrac{(s-c)(s-a),}{s(s-b)}}$

where $a = 120{\cdot}5$, $b = 91{\cdot}7$, $c = 127{\cdot}4$, and s is half the sum of a, b and c.

6. If W is the weight in pounds per 1000 yards of a wire whose diameter is D inches, then corresponding values of W and D for wires of a certain metal are given in the following table :—

D	..	·020	·024	·032	·040	·048	·056	·064
W	..	3·63	5·23	9·30	14·53	20·92	28·48	37·20

Plot W vertically, taking 1 inch to represent 10 lb. per 1000 yards, against D horizontally, taking 1 inch to represent ·01 inch.

Use your graph to find (1) the length of a coil of wire of ·036 in. diameter, which weighs 2 lb. 4 oz.; (2) the weight in kilogrammes per kilometre of a wire 1·25 mm. diameter, taking 1 kilogram = 2·2 lb., and 1 kilometre = 1100 yards.

7. A steeplejack is repairing one side of a square chimney. In the course of the work he is compelled, for various reasons, to return to the ground, and every time he ascends the ladder he rises 4 feet higher than he did the previous time; the first time he climbs to a height of 20 feet and the last to a height of 60 feet. When he has completed the work, how many separate ascents will he have made and what is the sum of the distances he has climbed ?

8. Solve the equation $ax^2 + bx + c = 0$.

Find the sum and also the sum of the squares of the roots of the equation $5x^2 - 13x + 7 = 0$.

Use your results to find a new equation whose roots are the above sum and sum of squares.

6b

1. (1) Simplify

$$\frac{3}{(x-1)(x+2)} - \frac{2}{(x+3)(x-1)} + \frac{1}{(x+2)(x+3)};$$

(2) Factorise the expressions :—

$$x^4 - 26x^2 + 25, \quad (2x - y - z)^2 - (x - 2y + z)^2.$$

2. Determine a and b so that

$$12x^4 - 31x^3 + 14x^2 + ax + b$$

may be divisible by $(x-1)^2$ without remainder; and factorise the quotient.

3. Solve the equations—

(i) $\dfrac{2}{x+3} + \dfrac{3}{x+4} - \dfrac{5}{x+5} = 0$;

(ii) $\frac{1}{2}x + \frac{1}{3}y = 3, \ \frac{1}{2}x^2 - \frac{1}{3}y^2 = 5.$

4. In Association Football the goal-average of a team is the number of goals scored by it divided by the number scored against it. Two teams A and B play a game and A wins by one goal to nothing. Before the game started their goal-averages were 1·5 and 1·6 and after it 1·6 and 1·5 respectively. Find the number of goals scored by each team before this game.

5. Without using the tables find the values of $(\frac{1}{8})^{-\frac{2}{3}}$, $\log (1 \div \sqrt[5]{100})$ and the number whose logarithm is -3.

Use the tables to find the value of

$$\frac{(37{\cdot}69)^2 \times \sqrt[4]{0{\cdot}0765}}{48{\cdot}57 \times 1{\cdot}543}$$

6. Find the sum of n terms of a Geometrical Progression whose first term is a and whose common ratio is r.

A man sets aside £50 at the end of the first year, 20% more at the end of the second year, and so on in Geometrical Progression. Find an expression for the total amount set aside at the end of n years, and calculate its value when $n = 10$.

7. A piece of wire 24 in. long is cut into two pieces $4x$ in. and $(24 - 4x)$ in. long respectively; each piece is then bent into the form of a square. Find an expression for the sum of the areas enclosed by these squares. Calling this sum A square inches, plot A against x (using any suitable scale) for values of x from 0 to 6 inclusive.

Find from your graph the values of x for which the area enclosed by the two squares is 27 sq. in.

8. (1) Simplify $\dfrac{4}{2 - 3\sqrt{5}} + \dfrac{7}{3 + \sqrt{125}}$, giving your answer with a rational denominator.

(2) If $x + y$ varies as $z^2 + z$, and $x - y$ varies as $z^2 - z$, and when $x = 1$ and $y = 2$, $z = -2$, find the values of x and y when $z = 3$.

7b

1. Lead weighs w lb. per cubic inch. A leaden pipe has internal and external radii a, b inches and length h inches: what is its weight?

If it be melted and recast into n equal spheres, give a formula for the radius of each sphere.

(The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

2. Put into factors each of the three expressions $(2x - y)^2 - z^2$, $(2x + z)^2 - y^2$, $(z - y)^2 - 4x^2$, showing that they have a common factor.

Simplify $\dfrac{1 + \left(\dfrac{1 - x}{1 + x}\right)^2}{1 - \left(\dfrac{1 - x}{1 + x}\right)^2} \times \dfrac{2x}{x + \dfrac{1}{x}}$.

3. A goods train starts from A at 11.30 a.m., travels at 20 miles an hour, stops for 15 minutes at B, 10 miles from A, and then proceeds at the former speed. A passenger train on a separate track leaves A at 12.10 p.m., and travels at 40 miles an hour running through B without a stop. Where and when does the latter train pass the former ?

4. The first two terms of a Geometrical Progression add up to 10, and the first three terms add up to 19. Show that there are two such progressions, and find them.

5. Solve the equations :—

(*a*) $\frac{20}{x} - \frac{1}{y} = 4$, $\frac{2}{x} + \frac{1}{y} = \frac{3}{xy}$;

(*b*) $(x - 1)(x + 3)(x - 7) = (x - 3)^2(x + 1)$;

(*c*) $2x^2 - 11x - 10 = 0$, giving the roots to two decimal places.

6. Plot the graphs of $4y = 3x + 5$ and $y = 2x - x^2$ from $x = -1$ to $x = +3$. By means of your graphs and a construction which must be carefully explained, determine, as accurately as you can, the values of x for which the value of y in the first graph exceeds the value of y in the second graph by 2.

7. Give a definition of $\log_{10}x$.

Prove from first principles that $\log_{10}6 - \log_{10}2 = \log_{10}3$.

Use logarithms to evaluate to three significant figures $a\sqrt{b/c^3}$, where $a = 47{\cdot}2$, $b = 0{\cdot}3413$, and $c = 0{\cdot}265$.

8. Given that $a : b = c : d$, prove that

(i) $\frac{3a - 2c}{5a + c} = \frac{3b - 2d}{5b + d}$, (ii) $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{a^2}{b^2}$.

The perimeter of a right-angled triangle is 4 times the shortest side. What is the ratio of the shorter to the longer of the two sides containing the right angle ?

8b

1. Divide $6x^4 - 7x^3 + 21x^2 - 11x + 15$ by $2x^2 - x + 3$;

Simplify

$$\frac{x-1}{x^2+5x+6} + \frac{4(2x+1)}{3x^2+8x+4} - \frac{2x-1}{3x^2+11x+6}.$$

2. Solve the equations :—

(i) $\frac{1}{5}(2x+1) - \frac{1}{6}(3x-1) = \frac{1}{10}$;

(ii) $2\left(\frac{x}{b} - \frac{y}{a}\right) = 3\left(\frac{x}{a} + \frac{y}{b}\right) = 6$;

(iii) $(x+2)^3 - (x-1)^3 = 117$.

3. One clock gains t mins. every hour and a second loses t mins. every hour. The clocks are together at 12 o'clock (noon); what is their difference x hours later ?

Express in terms of t the time elapsed when the first clock registers 12 o'clock (noon) of the next day, and if at that instant the second clock registers 10 minutes to 12, find the value of t in the form of a fraction.

4. State and prove a formula for the sum of n terms of an Arithmetical Progression of which the first term is a and the common difference is d.

At the side of a sloping road which descends 1 foot vertically for every 9 feet horizontally, thirty posts are set up, the distance between the centres of consecutive posts being 2 feet. The tops of the posts are on the same level and each post is driven 3 feet into the ground. If the post furthest up the road projects 3 feet above the ground, find the total length of the posts.

5. A circular garden path is gravelled at 2*s*. per square yard, the cost being £6 12*s*. Along both edges of the path is a stone edging, at 3*s*. per yard, the cost of this being £19 16*s*. Find the width and the inner radius of the path. $\left(\text{Take } \pi = \frac{22}{7}.\right)$

6. The time of sunrise and sunset on October 21st and at intervals of 10 days thereafter are as follows :—

	O. 21	O. 31	N. 10	N. 20	N. 30.	D. 10
Rises . .	6.35	6.53	7 10	7.27	7.43	7.55
Sets . .	4.55	4.35	4.18	4.4	3.54	3.50

	D. 20	D. 30	Ja. 9	Ja. 19	Ja. 29
Rises . .	8.4	8.8	8.5	7.59	7.46
Sets . .	3.51	3.57	4.9	4.22	4.40

Plot the times of mid-day (half-time from sunrise to sunset) for these dates, and draw a graph to show the time of mid-day for any date between October 21st and January 29th. (Take the origin to represent October 21st and 12 o'clock, 1 in. vertically to represent intervals of 5 mins., and 1 in. horizontally to represent intervals of 20 days.) On what date does mid-day differ most from 12 o'clock, and by how much ? On what dates does mid-day occur at (i) 12 o'clock, (ii) 11.45 ?

7. The gradient which the average touring car may be expected to climb at 20 miles an hour on top gear is given by the following formula :

$$G = \frac{6W}{\frac{CR}{D} - \frac{W + 10}{7{\cdot}5}}$$

where G = gradient (1 in G),
W = loaded weight (cwt.),
C = cubic capacity of engine in c.c.
D = diameter of driving wheel in mm.,
R = gear ratio.

(1) Calculate G (to nearest integer) if $W = 18$, $C = 1800$, $R = 4{\cdot}75$, $D = 710$.

(2) Express W in terms of the other letters.

(3) Calculate W if a hill of 1 in 15 i just climbed on top gear at 20 m.p.h., the values of C, D and R being as in (1).

8. Express the sum and the product of the roots of the equation

$$a(x^2 + ax + a^2) = b(x^2 + bx + b^2),$$

in terms of its coefficients.

In the equation $3x^2 + 16x + 15 = 0$ find the sum of the squares of the roots; and also the sum of their reciprocals. Form the equation whose roots are the above sums.

9b

1. Simplify

(i) $\dfrac{12}{7(2x-3)} - \dfrac{1}{x-1} + \dfrac{1}{7(x+2)}$;

(ii) $\left(x + \dfrac{1}{x}\right)^2 + \left(y - \dfrac{1}{y}\right)^2 - \left(\dfrac{x}{y} + \dfrac{y}{x}\right)\left(xy + \dfrac{1}{xy}\right)$

2. A number of two digits is such that the units' digit is double the tens' digit; when the digits are reversed the number is increased by 36. What is the number?

3. Solve the equations—

(i) $\dfrac{1}{x+2} - \dfrac{2}{x+6} = \dfrac{1}{2x+3} - \dfrac{3}{2(x+3)}$;

(ii) $ax - by = a^3 - 3ab^2$, $bx + ay = 3a^2b - b^3$;

(iii) $2x + 3y = 7$, $x^2 + 2xy - 8 = 0$.

4. If $x - y = 1$, prove that (*a*) $x^3 - y^3 - 3xy = 1$, and (*b*) $2x^2 + xy - 3y^2 = 5x - 3$.

If $a = p(q^2 + r^2)$, $b = q(p^2 + r^2)$, $c = (p + q)(pq - r^2)$ find the value of $\sqrt{(c - a + b)(c + a - b)}$ in terms of p, q and r.

5. Without using tables, find the value of

(1) $(16)^{\frac{3}{4}} \div (27)^{-\frac{2}{3}} \div 6^4$; (2) $\log_{10}(1 \div \sqrt[4]{1000})$;

(3) $\log_{10}(\cdot 001)$.

By means of the tables evaluate $\dfrac{(3{\cdot}172)^2 \times \sqrt{141{\cdot}86}}{(5{\cdot}362)^3}$.

6. Find the values of a, b, c which make the expression $a(x-1)(x+3)+b(x+3)(x-3)+c(x-3)(x-1)$ equal to $+3$, -1, $+1$ when x is equal to $+3$, $+1$, -3 respectively.

Draw the graph of the expression from $x=-4$ to $x=+4$, a, b and c having the values found above.

Find where the graph is cut by that of $y=2x+1$.

7. Two aeroplanes start at the same moment from points A and B 210 miles apart, the slower to fly from A to B and the faster from B to A. Their speeds differ by 10 miles an hour. If the slower plane takes 1 hour and 36 minutes to reach B after the moment when it meets the other aeroplane, find the speed of each aeroplane.

8. Find the values of x and y where

$$(1)\ \frac{3-2\sqrt{7}}{8+3\sqrt{7}}=x+y\sqrt{7};$$

$$(2)\ \sqrt{x}+\sqrt{y}=\sqrt{23+4\sqrt{15}};$$

$$(3)\ (x+\sqrt{y})(3-\sqrt{5})=8.$$

10b

1. Find the value of $\sqrt{\dfrac{13x^2-6ax+2a^2}{11x-3a}}$ correct to three decimal places, when $x=5$ and $a=-3$.

Simplify

$$\left(\frac{1}{3x-4}+\frac{1}{4x-3}\right)\times\left[\frac{3-\dfrac{19}{x+5}}{1-\dfrac{6}{x+5}}\right].$$

2. (i) If x^2+2x-1 is a factor of $x^4+x^3-9x^2-ax+6$, what is a and what are the other factors of the expression?

(ii) Factorise $(2x-y+1)^3-(x-2y+1)^3$.

3. Solve the equations :

$$(1)\ \frac{a(x-a)}{b} - \frac{b(x+b)}{a} = a + b\,;$$

$$(2)\ \frac{2x}{3x-2} - \frac{4}{x+1} + \tfrac{1}{3} = 0\,;$$

$$(3)\ 2x - y = 8,\ 6z + 4y = 9,\ x + z = 5.$$

4. The number of eggs that can be bought for a certain sum of money is increased by a dozen if the price is reduced a penny each, and is decreased by 8 if the price is raised a penny each. What is the price ?

5. Show that $\frac{1}{3}$, $-\frac{1}{2}$, $\frac{3}{4}$ are successive terms in a Geometric series.

Find the sum to 10 terms of the Geometrical Progression

$$3 + 2^{\frac{1}{4}}3^{\frac{3}{4}} + 2^{\frac{1}{2}}3^{\frac{1}{2}} + \ldots$$

using logarithms to evaluate your answer.

6. Prove from first principles that $\log_{10}x^3 = 3\log_{10}x$.

Evaluate $\sqrt[3]{\dfrac{(\cdot 101)^2 \times 125600}{(3\cdot 51)^2 \times 69}}$.

7. A small heavy sphere of lead is suspended by a fine thread and swings freely as a pendulum. The time of 20 swings is observed for different lengths of thread, with the following results :—

Length of thread in in.	8	12	22	32	48	62	72	83
Time of 20 swings in secs. . . .	9	11	15	18	22	25	27	29

Draw a graph showing the times taken by pendulums of different lengths to make 20 swings.

What is the length of a pendulum that beats (i) seconds, (ii) half-seconds ?

8. If $\dfrac{x}{a} = \dfrac{y}{b} = \dfrac{z}{c}$, prove that each fraction is equal to

$$\sqrt[n]{\frac{px^n + qy^n + rz^n}{pa^n + qb^n + rc^n}}.$$

Three numbers are in the ratios $2\sqrt{3} : 5 : 6$ and the sum of their squares is 292. Find the numbers.

11b

1. (i) If $y = \dfrac{3 - 2x}{x + 1}$ and $y = \dfrac{3z - 1}{z - 2}$, find x in terms of z;

(ii) Find the square root of

$$x^6 + 6x^5 + x^4 - 20x^3 + 28x^2 - 16x + 4.$$

2. Factorise and find the L.C.M. of the following expressions :—

$$x^2 + 10x - 24,\ 3x^2 - 10x + 8,\ 4x^2 + 51x + 36.$$

3. Simplify $\dfrac{x + 1}{x^2 - 4x + 3} - \dfrac{x - 3}{x^2 - 1}$.

Solve the equations :—

(i) $\dfrac{2x - 3}{x - 1} - \dfrac{x - 7}{x + 2} = 2\frac{1}{3}$;

(ii) $2x = 2{\cdot}1 + 3y,\ 3(x + y - {\cdot}2) = -y.$

4. A man walks a certain distance. Had he walked half the distance further at the same rate he would have taken 40 minutes longer, and had he walked at the rate of half a mile per hour slower he would have taken 16 minutes longer to cover the original distance. Find the distance and the time of walking.

5. The fifth term of an Arithmetical Progression is 33, and the seventeenth term is 60. Find the first term and the common difference.

Which term of the series has a value nearest to, but less than, 100 ?

What is the sum of the series up to and including this term ?

6. Draw the graphs of $y = 3x^2$ and $2y - x = 4$ between $x = 3$ and $x = -3$. Hence solve the equation

$$6x^2 - x - 4 = 0.$$

Also solve the equation by another method.

7. Evaluate, without using the tables, (i) $\log_{10} 10^{4\cdot72}$; (ii) $100^{\overline{2}\cdot5}$; (iii) $(1\frac{25}{144})^{\frac{1}{2}}$.

Use tables to evaluate $a^2b^3c^{-\frac{1}{2}}$ when $a = 24\cdot67$, $b = 0\cdot0426$, $c = 0\cdot007843$, giving your answer correct to 3 significant figures.

8. If $a + \sqrt{b} = c + \sqrt{d}$, where a, c are rational and $\sqrt{b}$, $\sqrt{d}$ are surds, show that $a = c$ and $b = d$.

If $\sqrt{5(31 - 8\sqrt{15})} = \sqrt{x} - \sqrt{y}$, find x and y.

12b

1. Simplify $\dfrac{x+3}{x^2+x-2} - \dfrac{2(x-2)}{x^2-4x+3} + \dfrac{2x-1}{x^2-x-6}$,

and verify your result by putting $x = 4$.

2. Find the factors of—

(i) $3ax - 6 + 4a^2x - 8a$;

(ii) $96x^2 + 4x - 1$;

(iii) $(x-3)^2 - 2(x-3)(y+2) - 3(y+2)^2$.

3. Two towns A and B are 20 miles apart. A man takes 11 hours 40 mins. to walk from A to B and back again to A, his rate on the return journey being one mile an hour slower than on the outward journey. What was his rate of walking in the outward journey?

4. In an examination in Arithmetic the percentages (y, z) of boys and girls respectively who obtained x per cent. or more of maximum marks are given for certain values of x by the table:—

$x =$	10	20	30	40	50	60	70	80	90	100
$y =$	94	83	70	56	43	31	20	10	3	0
$z =$	89	74	58	43	30	19	11	5	1	0

Draw the graphs of y and z in the same diagram with the same axes, taking the x-axis horizontally and a scale of 1 in. to 20.

What percentage (i) of boys, (ii) of girls, obtained a third or more of maximum marks? What was the

median mark (i) for boys, (ii) for girls ? (By the median mark is meant that which is below the mark of just as many candidates as it is above the others.)

5. A rod is to be cut into five parts whose lengths are in geometrical progression, and the lengths of the shortest and longest parts are to be 6 inches and 24 inches respectively. Find the length of the rod required.

6. Two numbers differ by four. The sum of their squares exceeds their product by the same amount as 100 exceeds the smaller number. What are the numbers ?

7. If α and β are the roots of $ax^2 + 2bx + c = 0$, prove that $\alpha + \beta = -\frac{2b}{a}$ and $\alpha\beta = c/a$.

The Arithmetic mean of the roots of a certain equation is $\frac{7}{3}$, and the Arithmetic mean of their reciprocals is $\frac{7}{5}$; find the equation in its simplest form.

8. (i) If $a : b = c : d$, show that $ab + cd$ is a mean proportional between $a^2 + c^2$ and $b^2 + d^2$.

(ii) R, the electrical resistance of a wire, varies as the length of the wire and inversely as the square of the diameter. If d is the diameter and W is the weight, show that $R \propto W/d^4$.

A wire has a diameter 0·15 cm. and its resistance is one-fifth of an ohm. Find, as a decimal correct to the third place, the resistance of a wire of the same weight and of the same material but having a diameter 0·24 cm.

13b

1. (i) Simplify

$$\frac{(ax - b)^2 - (bx - a)^2}{a^2 - b^2} \times \left[\frac{a + b}{x + 1} - \frac{a - b}{x - 1}\right];$$

(ii) Multiply $3x^3 - 4x^2 + 9x - 2$ by $4x^2 - 8x + 3$.

2. Resolve into factors and find the H.C.F. of the following expressions: $x^2 + 7x - 18$, $(x - 3)(x^2 - x) + 6(x - 3)(x - 6)$, $2x^2 - 27a - 3ax + 18x$.

3. Solve the equations—

(i) $\dfrac{2x - 3}{11} - (4x - 11\frac{1}{6}) + \frac{1}{5}(7x + 8) = 0$;

(ii) $4x^2 - 7x - 33 = 0$ to 2 decimal places;

(iii) $\cdot 3x - \cdot 2y = 3\cdot 3$, $9x + 10y = xy$.

4. A man left £79,200 to be divided equally among his relatives surviving at his death. Four of the possible heirs died before the man, so that each of the remaining relatives received £800 more than they would have done had all survived. How much did each surviving relative receive?

5. Prove a formula which gives the sum of n terms of the series

$$a + (a - d) + (a - 2d) + (a - 3d) + \ldots$$

If the 6th term of this series is 102, and the 16th term is 78, find the first term, the common difference, and the sum of all the terms which are positive.

6. A plane cuts off from a sphere of radius 10 inches a segment of height h inches. What fraction of the sphere is cut off if h is 10? The fraction cut off in general is given by the function

$$\tfrac{1}{4}\left(\frac{h}{10}\right)^2\left(3 - \frac{h}{10}\right).$$

Plot horizontally this function against h for $h = 0, 2, 4, 6, 8, 10$. (Represent 2 by 1″ horizontally and represent 0·5 by 5″ vertically.)

What is the height of the segment if the volume is $\frac{1}{4}$ of that of the whole sphere?

7. Prove from elementary principles that

$$\log_{10} x^5 = 5\log_{10} x.$$

Evaluate, by means of logarithm tables,

$$\frac{(3{\cdot}715)^2 \times \sqrt{{\cdot}0178}}{({\cdot}9617)^3}.$$

8. Simplify

$$\frac{3\sqrt{5} - \sqrt{6}}{\sqrt{5} - \sqrt{2}} \times [3\sqrt{10} + \sqrt{30} + 2\sqrt{3} + 15].$$

14b

1. (i) In each of the formulæ (*a*) $Pt = m(v - u)$, (*b*) $Ps = \frac{1}{2}m(v^2 - u^2)$, express v in terms of the other letters.

(ii) Find the factors of (*a*) $x^2 + 14x - 72$, (*b*) $x^2 + x(a + 6) - 2a^2 + 3a + 9$.

2. Solve the equations :—

(i) $\dfrac{2a}{a - 2x} - \dfrac{a}{2a - x} = \dfrac{2x}{x - 2a} - \dfrac{x}{2x - a}$;

(ii) $x + 3y - z = \frac{1}{2}$, $3x + 2z = 7$, $y + 4z = \dfrac{5}{3}$;

(iii) $3x^2 - 4x + {\cdot}56 = 0$. (Correct to 2 decimal places.)

3. A man buys fresh eggs ; the number of dozen he buys is 2 less than three times the price, in shillings, of the eggs per dozen. He pays 13*s*. 9*d*. How many eggs does he buy and what is the price of the eggs per dozen ?

4. Simplify (i) $(81)^{-\frac{1}{4}} \times (54)^{\frac{1}{3}} \times (\frac{2}{3})^{-\frac{3}{2}}$; (ii) $10^{\bar{1}{\cdot}7820}$; (iii) $10^{\log_{10}2}$.

(Logarithm tables must be used only in (ii)).

By means of tables, evaluate

$$\frac{a^{\frac{2}{5}}}{bc^3} \text{ when } a = 0{\cdot}0482,\ b = 17{\cdot}84,\ c = {\cdot}425.$$

5. Assuming the formula for the sum of n terms of a Geometrical Progression whose first term is a and whose

common ratio is r, prove that the ratio of the sum of $2n$ terms to that of n terms is $r^n + 1$.

Find a similar expression for the ratio of the sum of $3n$ terms to that of n terms.

The sum of the 9th, 10th and 11th terms of a G.P. is 11·7, the tenth term being 3. What is the common ratio ?

6. A lady spends a certain sum in buying material by the yard. If the price had been $1\frac{1}{2}d.$ per yard dearer she would have had 1 yard less for the same money; if the price had been $1d.$ per yard cheaper she could have received 1 yard more by spending $4\frac{1}{2}d.$ more than she did. Find the length of material bought and the price per yard.

7. Plot, on the same axes, the graphs of $8y = 7x^2 - 12x - 11$ and $4y = 3x - 1$. From your graph find the co-ordinates of the points where the two graphs cut.

8. (i) Solve the equation $\sqrt{4x+1} - \sqrt{2x+1} = 2$.

(ii) Three metal balls of radii $\frac{1}{2}$ in., $1\frac{1}{2}$ in. and 2 in. are melted down and formed into a single ball. Given that the volume of a sphere is proportional to the cube of its radius, find the radius of the new ball, correct to three significant figures.

15b

1. By means of the Remainder Theorem, or otherwise, find for what values of b and c the expression $2x^3 + bx^2 - 8x + c$ is exactly divisible by $x^2 - 2x - 3$.

Simplify $\dfrac{3x-7}{x^2-5x+6} - \dfrac{5}{x^2+x-6} - \dfrac{x+9}{x^2-9}$.

2. Divide

$3x^5 + (3c+1)x^4 + 36x^3 - c^2x^2 - 4(1+3c)x + 4c$

by $3x^2 + x - c$.

Factorise the following :—

(i) $(2x^2 + a^2 + b^2 + 2ax + 2bx) - 2(a+x)(b+x)$;

(ii) $(x - y + 1)^3 + (2x + y - 3)^3$.

3. Solve the equations :—

$$(1)\ \frac{x+3}{3}+\frac{1}{x+2}=\frac{x+4}{2}-\frac{x}{6};$$

$$(2)\ 4x-2y=17,\ 2x^2+2y^2=37.$$

4. ABC is a right-isosceles triangle right-angled at A; on AC as base another right-isosceles triangle is drawn (outside ABC) with the right angle at D; similarly another, ADE, is drawn outside ACD with right angle at E. If the length of AB is 10 inches find the length of each of the short sides of the eighth triangle so drawn (including ABC), and find also its area. Find the sum of the area of the 8 triangles.

5. Evaluate, by means of tables, $\sqrt[3]{\dfrac{\cdot0423\times(4\cdot562)^2}{(\cdot862)^5\times 17610}}$.

6. Two numbers are such that their difference is 21. The square of their sum exceeds 7 times their product by $\frac{5}{3}$ of their sum. Find the numbers.

7. Plot the graph of the function $3-5x-x^2$ when x is between -7 and $+3$; find its maximum value and the values of x between which it is positive.

8. Express in terms of a, b and c (i) the sum, (ii) the product and (iii) the difference of the roots of the equation

$$ax^2+2bx+c=0.$$

Find the equation whose roots are the reciprocals of the roots of

$$3x^2-5x-11=0.$$

If α, β are the roots of the last-given equation, find the equation whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$.

R. B. A. N. M'S EDUCATIONAL CHARITIES.
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GEOMETRY

1c

1. State and prove the theorem relating to the sum of the interior angles of a polygon of n sides.

Given that the interior angle of a regular polygon is 140°, find the number of sides.

A square $BDEC$ is constructed on the side BC of an equilateral triangle ABC. Calculate the angle AEC.

2. Prove that the opposite sides and angles of a parallelogram are equal and that the diagonals bisect each other.

$ABCD$ is a parallelogram. Prove that the lines joining A and C to the mid-points of CD and AB respectively trisect the diagonal BD.

3. Prove that, in an acute-angled triangle, the square on the side opposite the acute angle is equal to the sum of the squares on the other two sides minus twice the rectangle contained by one of them and the projection on it of the other.

Calculate the length of the longer diagonal of a rhombus with an angle of 120° and sides of length x inches.

4. Prove that the rectangles contained by the segments of intersecting chords of a circle are equal, whether they intersect inside or outside the circle.

A point P is taken distant 5 ins. from the centre of a circle of radius 3 ins. A circle, centre P, and radius 3 ins., is drawn to cut the first circle at A, and PA is

produced to cut the first circle again at B. Calculate the length of AB.

5. Construct a quadrilateral $ABCD$ from the following data :—AB and AD are respectively equal to 2·1 and 2·6 ins., and angles ABC, BCD and CDA are respectively equal to 75, 85 and 105 degrees.

Prove that a circle can be drawn through the corners of the quadrilateral and show how to find its centre.

6. Show how to describe a triangle equal in area to any given quadrilateral. Show also how to find the length of a side of a square equal in area to this triangle.

7. Prove that the common chord of two intersecting circles bisects each of their common tangents, and is bisected by the line joining the centres.

8. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

The sides AB, AC of a triangle ABC are 10″ and 6″ respectively. CD is a median and is met at P by the bisector of the angle A. Given that the area of the triangle ABC is 40 sq. ins., find the area of the triangle APC.

2c

1. If two angles of a triangle are equal, prove that the sides opposite them are equal.

If one of the diagonals of a parallelogram bisects the two opposite angles at its ends, prove that the diagonals bisect each other at right angles.

2. Prove that, if two sides of a triangle are unequal, the greater side has the greater angle opposite to it.

Deduce that the perpendicular is the shortest of all straight lines than can be drawn to a given straight line from a given point outside it.

3. Draw a quadrilateral $ABCD$ in which $AB = 1{\cdot}6''$, $AC = 2''$, angle $BAC = 70°$, $BD = 3{\cdot}4''$, and the diagonal AC bisects the diagonal BD.

Now construct a rhombus equal in area to $ABCD$ and having AC as one diagonal. Prove your construction to be correct.

4. Any line drawn from a point A outside a circle cuts it at B and C. AT is a tangent from A. Prove that the square on AT is equal to the rectangle contained by AB and AC.

Find the locus of the point of contact of tangents drawn from a point A on a straight line ABC to the circles which can be drawn through the points B and C.

5. If a straight line touch a circle and from the point of contact a chord be drawn, prove that the angles which this chord makes with the tangent are equal to the angles in the alternate segments. Two straight lines ACD, BCE intersect at C and AB is parallel to ED; prove that the circumcircles of the triangles ABC CDE touch.

6. If the angle A of a triangle ABC is 60°, show that the sum of the squares on AB, AC is equal to the square on BC together with the rectangle AB, AC.

If $AB = 7$, $AC = 8$, and angle $A = 60°$, find the length of the median through A, (i) by drawing, (ii) by calculation.

7. Prove that the opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles, including the case when the centre is outside the quadrilateral.

The vertex A of an equilateral triangle is joined to any point D in the base BC produced and an equilateral triangle ADE is described on AD, E and C being on opposite sides of AD. Prove that the perpendicular bisectors of AC, AD and AE are concurrent.

8. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on corresponding sides.

The triangle ABC is right-angled at A and AD is perpendicular to BC. Prove that the ratio of BD to DC is equal to the square of the ratio of AB to AC.

3c

1. Prove that the sum of the angles of a triangle is equal to two right angles.

From the vertex A of a triangle ABC with an obtuse angle at B a perpendicular AD is drawn to the opposite side; prove that B is between C and D.

2. A straight line $PQRS$ cuts two parallel straight lines AB and CD in Q and R respectively. Write down one pair of each of (i) alternate angles, (ii) corresponding angles, (iii) vertically opposite angles and (iv) conjoined angles. State how the two angles forming each pair are related to each other.

Prove that if the lines which bisect the opposite angles A and C of a quadrilateral $ABCD$ are parallel to one another, the angles B and D are equal.

3. Prove that, in any right-angled triangle, the square on the side opposite the right angle is equal to the sum of the squares on the two sides containing the right angle.

4. Construct a quadrilateral $ABCD$ such that $AB = 2{\cdot}3''$, $BC = 1{\cdot}7''$, $ABD = 60°$, $ABC = 135°$ and $BCD = 75°$. Then find the length of the side of a square whose area is equal to that of the quadrilateral.

5. Prove that the straight line joining the middle points of two sides of a triangle is parallel to the base and equal to half the base.

P is a variable point on a fixed line XY and A is a fixed point (not on XY); find the locus of the mid-point of AP.

6. Prove that angles in the same segment of a circle are equal. Deduce that the angle in a semicircle is a right angle. Also prove this by an independent method.

7. On a base BC of length 4·5 cm. it is required to construct a triangle ABC with the angle A equal to 75°. Draw the locus of A.

If the angle B is twice the angle C proceed to find the position of A.

8. Show that the corresponding sides of equi-angular triangles are proportional.

ABC is a triangle right-angled at A ; X is any point on BC, and XY and XZ are perpendiculars to AB and AC respectively ; given that AC, XY and XZ are equal to 8, 5 and 2 respectively, find AB.

4c

1. $ABCD$ is a parallelogram. On the opposite sides AB, CD equilateral triangles APB, CSD are drawn external to the parallelogram, and on BC an equilateral triangle BQC is drawn on the same side of BC as the parallelogram. Prove that PQ and QS are equal to the diagonals of the parallelogram.

2. Through a number of equally spaced points on a straight line, parallel straight lines are drawn. Show that equal lengths are intercepted by these parallels on any straight line that cuts them.

Through C, the mid-point of the base AB of a triangle OAB, a straight line PCQ is drawn, meeting OA in P and OB produced in Q. Prove that CQ is greater than CP. (Through A draw a line parallel to BO.)

3. Prove that the area of a parallelogram is equal to the area of the rectangle on the same base and between the same parallels.

In a parallelogram $ABCD$ the base BC is 5 cm., the angle ABC is 60° and the altitude is 3·6 c.m. Draw the parallelogram, calculate its area and the length of the side AB, and also measure this side from your figure.

4. Prove that a triangle is isosceles if two of its angles are equal. State and prove the converse of this theorem.

5. If the opposite angles of a quadrilateral are supplementary, prove that the quadrilateral is cyclic.

Equilateral triangles ABD, ACE are described on the sides AB, AC of a triangle ABC, so that D and E fall

outside the triangle; BE, CD intersect at H. Prove that A, E, C, H lie on a circle.

6. Prove that the internal bisectors of the angles of a triangle are concurrent at a point which is the centre of the inscribed circle of the triangle.

7. If a straight line touch a circle and from the point of contact a chord be drawn, prove that the angles which the chord makes with the tangent are equal to the angles in the alternate segments.

Two unequal circles cut at A and B. The tangents at A to the circles cut them again at P, Q. PQ, produced if necessary, cuts the circle AQB at S, and the circle APB at T. Prove that AS is equal to AT.

8. Prove that the internal bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.

ABC is a triangle, D the middle point of the side BC; AD is joined and the angles ADC, ADB are bisected by straight lines which meet CA, AB, in E F respectively; prove that FE is parallel to BC.

5c

1. Prove that all the interior angles of a convex polygon, together with four right angles, are equal to twice as many right angles as the polygon has sides.

The angles of a hexagon are in the ratio 3 : 5 : 7 : 9 : 11 : 13; find them.

2. Prove that two triangles are congruent if the three sides of one are respectively equal to the three sides of the other.

Show that a quadrilateral which has its opposite sides equal must be a parallelogram.

3. Construct a quadrilateral $ABCD$ with the side DC parallel to AB and at a distance 1·2 in. from it, the lengths of the sides AB, BC and AD being respectively 5 in., $1\frac{1}{2}$ in., and 2 in., and the angles A and B both acute.

By measuring any sides or lengths and making any

further constructions you choose, find the area of $ABCD$.

4. Prove that in an obtuse-angled triangle, the square on the side opposite the obtuse angle is equal to the sum of the squares on the sides containing it plus twice the rectangle contained by one of those sides and the projection on it of the other.

ABC is a triangle having the angle A obtuse, and the side AC is 2 in., BC is twice AB, and the projection of AB upon AC is one quarter of AB. Calculate the lengths of the sides of the triangle.

5. Prove that angles in the same segment of a circle are equal.

The opposite angles at B and D of a cyclic quadrilateral $ABCD$ are bisected by lines meeting the circumference at P and Q. Prove that PQ is a diameter of the circle.

6. Prove that, if two tangents are drawn to a circle from an external point, the tangents are equal and the line joining the centre to the external point bisects at right angles the line joining the points of contact of the tangents.

7. If two chords of a circle (produced if necessary) cut one another, the rectangle contained by the segments of the one is equal to the rectangle contained by the segments of the other. Prove this.

8. Prove the following construction for inscribing a square in a given triangle ABC. "On BC describe the square $BXYC$ outside the triangle; join AX and AY, cutting BC in X' and Y' respectively; then X' and Y' are two corners of the square." Complete the construction.

6c

1. A tree is equidistant from two straight paths AB, AC which intersect at an angle of 30°, and is 70 yards distant from a third straight path which crosses the

other two at B and C. AB is 500 yards and AC 250 yards. Find all the possible positions of the tree and their distances from A.

2. If two angles of a triangle are equal to one another, prove that the sides which are opposite to the equal angles are equal to one another.

Prove that the median AD of a triangle ABC is less than half the sum of the sides AB, AC; and that the sum of the medians of a triangle is less than the sum of the sides.

3. Prove that a parallelogram is bisected by either diagonal.

$ABCD$ is a parallelogram, E a point in the diagonal AC, FEG a line parallel to AB cutting AD and BC in F and G respectively, and HEK a line parallel to BC cutting AB and CD in H and K respectively. Show that the area of the quadrilateral $FHGK$ is half that of $ABCD$, and that the parallelograms $BGEH$, $DFEK$ are equal in area.

4. AD is the perpendicular from a point A on a line BC; prove that the difference of the squares on AB and AC is equal to the difference of the squares on BD and CD.

If AB, AC and BC are 7 in., 5 in. and 6 in. respectively, find BD, DC, AD and the area of the triangle ABC.

5. Draw and explain a figure to illustrate the algebraical identity

$$(a - b)^2 = a^2 - 2ab + b^2.$$

A straight line AB is divided equally at O and unequally at P; prove that the sum of the squares on AP and PB is double the sum of the squares on AO and PO.

6. If two circles touch each other externally, prove that their centres and the point of contact are in one straight line.

ABC is a triangle. With centres A, B, C three equal

circles are drawn, no two of which intersect or touch each other. State fully how you would find, by geometrical construction, the centre and radius of a circle which would touch all three circles externally.

7. P is a point on an arc of a circle, whose chord is AB. AP is produced to Q, so that PQ is equal to PB. Prove that (i) the angle AQB is of the same size for all positions of P, (2) the locus of Q is an arc of a circle.

The perpendicular bisector of AB meets the original arc in O; prove that O is the centre of the circle on which Q lies.

8. If a straight line is drawn parallel to one side of a triangle, prove that it divides the other sides (produced if necessary) proportionally.

Two triangles ABC, ABD have a common side AB, C and D being on opposite sides of AB, and the angles ABC, ABD are obtuse; AB is produced to meet CD in O. Prove that the ratio of the areas of the triangles ABC and ABD is equal to the ratio of CO and OD.

7c

1. Prove that the opposite sides and angles of a parallelogram are equal.

In a parallelogram $ABCD$ the side AB is greater than the adjacent side AD; prove that the point E where the bisector of the angle A cuts the side DC is between C and D. If AE is produced to meet BC produced at F, show also that CE is equal to CF.

2. On a line AB, 2·5 centimetres long, draw a square $ABCD$. On the diagonal AC as base construct a triangle ACE equal to the square in area and having the angle ACE equal to 125°. State the construction clearly and prove it to be correct.

3. ABC is a triangle right-angled at A; $BCDE$, $ABHK$ are squares described on BC, AB respectively, outside the triangle; AL is drawn perpendicular to ED,

meeting BC in M. Show that the rectangle $BC.BM$ is equal to the square on AB.

4. AB, CD, EF are three parallel straight lines; ACE is a straight line perpendicular to the three, and $AC = CE = 2$ in. B, D, F are also in a straight line and $BD = 3$ in. Prove by elementary geometry that DF also equals 3 in.

State the general proposition of which the above is a particular case.

5. Prove that the angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

A, B, C are three points on a circle such that ABC is an acute-angled triangle; BQ and CR are diameters of the circle, and AQ, AR are joined. Show that the angles BAR, CAQ are equal.

6. If a straight line touches a circle and, from the point of contact, a chord is drawn, the angles which the chord makes with the tangent are equal to the angles in the alternate segments of the circle. Prove this.

A circle is inscribed in a triangle ABC, the points of contact with the sides BC, CA and AB being P, Q and R respectively. What is the relation between the angle BAC and the angle RPQ?

7. I is the centre of the inscribed circle of a triangle ABC, and J is the centre of the escribed circle which touches BC, and touches AB, AC produced. Prove that the circle on IJ as diameter passes through B and C.

The angle A of a triangle ABC is 56° and the radius of its inscribed circle is 1·5 cm. The distance from I to J (as above) is 6·2 cm. Construct, full size, the triangle ABC and measure the side BC.

(All the essential steps of the construction must be stated. Angles and lines may be bisected respectively by protractor and ruler if desired.)

8. If two triangles have an angle of one equal to an

angle of the other, and the sides about these angles proportional, the triangles are equi-angular. Prove this.

A quadrilateral $ABCD$ is such that the product of AB and AD is equal to the product of BC and CD, and the angle ABC is equal to the angle ADC. Prove that opposite sides of the quadrilateral are equal.

8c

1. Prove that, if one side of a triangle is produced, the exterior angle is equal to the sum of the two interior opposite angles.

Deduce the theorem relating to the sum of the three angles of a triangle.

P is any point inside a triangle ABC; prove that angle BPC is greater than angle BAC.

2. Prove that two right-angled triangles are congruent if the hypotenuse and a side of one are respectively equal to the hypotenuse and a side of the other.

AB, CD are two equal straight lines such that the line joining B and C is perpendicular to AB and CD, A and D being on opposite sides of BC; O is the mid-point of BC. Prove that AD passes through O.

3. Prove, without assuming the formula for the area of a rectangle or triangle, that a parallelogram is equal in area to the rectangle on the same base and between the same parallels.

PQ, RS are the parallel sides of a trapezium $PQRS$. PQ is less than RS, and the non-parallel sides PS, QR are equal. Prove the truth of the following construction for a rectangle $ABCD$ equal in area to $PQRS$ and having one side equal to PQ:—"Draw PB, QC perpendicular to SR; draw RA parallel to PC to cut BP produced at A; draw AD parallel to PQ to cut CQ produced at D."

4. Show that, if the square on one side of a triangle is equal to the sum of the squares on the other two sides, the triangle is right-angled.

Prove that a triangle whose sides are $(m^2 - n^2)$ in., $(m^2 + n^2)$ in., and $2mn$ in., is necessarily right-angled.

5. Prove that any two sides of a triangle are greater than the third side. Deduce that the shortest and longest distances from a point to a circle lie along the diameter through the point.

6. Define a cyclic quadrilateral. State and prove a property of its angles.

AOB is a diameter and AS a chord of a circle whose centre is O. The chord AP bisects the angle BAS. AS is produced to Q so that PQ is equal to PA. Prove that (i) OP is parallel to AQ; (ii) angles APQ, BPS are equal; (iii) angle SPQ is a right angle; (iv) triangles APB, QPS are congruent.

7. Prove that the locus of a point, which is equidistant from two given points, is the perpendicular bisector of the straight line adjoining the given points.

Show how to circumscribe a circle to a triangle. If O is the centre of the circle circumscribed to a triangle ABC, prove that the angle OBC is the complement of the angle BAC.

8. If the three sides of one triangle are proportional to the sides of another triangle, prove that the triangles are equi-angular.

Prove that if two quadrilaterals have their sides proportional taken in order, and a pair of corresponding angles equal, their remaining angles are equal in order.

9c

1. State and prove a formula for obtaining the size of an internal angle of a regular polygon of n sides.

A square $ABCD$, and a regular pentagon $ABPQR$ are described on opposite sides of a line AB. By calculating the sizes of the angles RAD, PRD (not by drawing to scale), prove that one of these angles is double the other.

2. Prove that the opposite sides and angles of a

parallelogram are equal, and that the diagonals bisect each other.

The acute angle between the diagonals of a parallelogram is 52° ; the side of the parallelogram opposite this angle is 4·5 cm. ; and one of the diagonals is 11·2 cm. Construct the parallelogram full size, stating the steps in your construction.

3. Prove that the square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.

If $ABCD$ is a square, and AL, BM are drawn perpendicular to any line through D, and AN is perpendicular to BM or BM produced, prove that $ALMN$ is a square.

4. A, B are two points 3·4 inches apart and BC is a line such that angle ABC is equal to 55°. By geometrical constructions and measurement find the distances from A of (1) a point P in BC which is equidistant from A and B ; and (2) a point Q in BC which is 1·2 inches nearer B than A. The steps in the constructions must be stated.

5. Prove that equal chords of a circle are the same distance from the centre.

A rectangular piece of cardboard, 4 in. by 3 in., is fitted into a hemispherical bowl of diameter 13 in. ; find the distance of the intersection of the diagonals from the centre of the bowl, explaining your method.

6. Prove that the angles made by a tangent to a circle with a chord drawn from the point of contact are respectively equal to the angles in the alternate segments of the circle.

The diameter AB of a circle is produced to a point C such that the length of the tangent CP from C to the circle is equal to the chord PA. Prove that BC is equal to the radius of the circle.

7. OA is perpendicular to OB, and ON is perpendicular to AB. Points C, D are taken in OA, OB respectively

so that CD is bisected by ON. Prove that $ABCD$ is a cyclic quadrilateral.

8. Prove that, if the vertical angle of a triangle is bisected internally or externally by a straight line which cuts the base, or the base produced, it divides the base internally or externally in the ratio of the other sides of the triangle.

10c

1. Construct a triangle ABC having the side AB equal to 2 ins., the side BC equal to 1 in., and the angle C a right angle.

Prove that the angle A is 30°.

2. Define parallel lines, a parallelogram, a rectangle and a rhombus. Show that diagonals of a rhombus bisect each other at right angles.

3. Prove that, in any triangle, the sum of the squares on two sides is equal to twice the square on half the base plus twice the square on the median which bisects the base.

Deduce that the sum of the squares on the sides of a parallelogram is equal to the sum of the squares on the diagonals.

4. Construct a quadrilateral $ABCD$ with diagonals intersecting at O, such that AB, BC, BD and AC are equal to 2·2, 2·7, 3·3 and 4 ins. respectively, and angle AOB is equal to 106°. Measure angle ADC.

5. Draw and explain a figure illustrating the algebraic identity $(a + b)^2 - (a - b)^2 = 4ab$.

If a line AB is bisected at C and produced to D, prove that the difference of the squares on AD and BD is four times the rectangle contained by AC and CD.

6. Prove that angles in the same segment of a circle are equal, proving any proposition you quote concerning angles subtended by arcs of a circle.

AB is a diameter of a circle whose centre is O, and P

is any point inside the circle. PN is drawn perpendicular to AB, and AP, BP are joined and produced to cut the circle again at C and D respectively. Prove that PN bisects the angle DNC, and that D, O, N, C lie on a circle.

7. State precisely, but without proof, how you would find the centres of (1) the circumscribed circle of a triangle; (2) a circle touching all three sides of a triangle; (3) a circle touching one side of a triangle and the other two sides produced.

Prove that the radius of the circle drawn touching one side of an equilateral triangle and the other two sides produced, is equal to the sum of the radii of the circumscribed and the inscribed circles of the triangle.

8. If ABC is a triangle with a right angle at C show that AC is a mean proportional between AB and the projection of AC on AB.

If from any point P on the circumference of a given circle a perpendicular PN is drawn to the tangent at another point A, show that AP^2/NP is constant.

11c

1. O is the intersection of the diagonals AC, BD of a square $ABCD$. POR, QOS are any two lines at right angles to each other, cutting AB, CD at P and R respectively, and BC, AD at Q and S respectively. Prove that $OP = OQ$, and that $PQRS$ is a square.

2. If two triangles have three sides of the one equal to three sides of the other, each to each, prove that they are equal in all respects.

BAC is any angle, P and M are points on AB and Q and N are points on AC such that AP is equal to AQ and AM is equal to AN. If PN, QM cut at S prove that AS bisects the angle BAC.

3. Prove that, in any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing it minus twice the rectangle

contained by one of those sides and the projection on it of the other.

By considering an obtuse-angled triangle and applying the above to one of the acute angles in it, deduce the corresponding theorem for the case of an obtuse angle.

4. Prove that the angle at the centre of a circle is double the angle at the circumference standing on the same arc, whatever be the size of the angle at the centre.

P, Q, R, S are four points on the circumference of a circle such that another circle can be inscribed within the quadrilateral $PQRS$. Prove that the two lines which join the points of contact of opposite sides are perpendicular to each other.

5. A quadrilateral is such that its diagonal AC bisects it. Draw the quadrilateral, full size, when AB, BC, CA are equal to 9·6 cm., 4·2 cm., and 8·8 cm. respectively, and angle BCD is equal to 128°, stating the steps in your construction. (The construction must not involve the calculation of any areas.)

6. Prove that the tangents drawn to a circle from an external point are equal.

Prove that the opposite sides of a quadrilateral circumscribed to a circle subtend supplementary angles at the centre.

7. O is the centre of a circle of radius 6 cm., and A, B are two points on the circumference such that angle $AOB = 50°$. Construct a circle to touch OA, OB and the arc AB, stating the steps in your construction and proving its truth.

8. If a straight line is drawn parallel to one side of a triangle, prove that it divides the other two sides (produced if necessary) proportionally.

P and Q are points in the sides AB, AC respectively of a triangle ABC. $AP = PB$ and $AQ = 2QC$. Prove that BQ and PC intersect at a point O such that BO is three times OQ. (Join AO and compare triangles.)

12c

1. Prove that if one side of a triangle is produced, the exterior angle is greater than either of the interior opposite angles.

Prove that, if one straight line cuts two other straight lines such that the alternate angles are equal, the two straight lines are parallel.

2. Prove that triangles on the same base and between the same parallels are equal in area, but that, if a triangle and a parallelogram are on the same base and between the same parallels, the area of the parallelogram is double that of the triangle.

3. Prove that, if two angles of a triangle are unequal, the greater angle has the greater side opposite to it.

If $ABCD$ is a parallelogram prove that the diagonal AC makes a greater angle with the shorter side through A than with the longer (use the converse of the above theorem.)

4. Prove that a line, drawn from the centre of a circle perpendicular to a chord, bisects the chord.

O is the centre of a circle of radius 6·5 cm., and P is a point 5·5 cm. distant from O. A chord AB is drawn through P so that BP is three times AP. Find the length of the chord.

5. Prove that the three altitudes of a triangle are concurrent.

If the three altitudes of the triangle ABC are AD, BE and CF, and H is the point in which they meet, prove that AD bisects the angle EDF. What particular point is H in connection with the triangle DEF ?

6. If from any point outside a circle, a secant and a tangent are drawn, prove that the rectangle contained by the whole secant and the part of it outside the circle is equal to the square on the tangent.

B, E are the opposite ends of a diameter of a circle. A, D are points on the circle on opposite sides of BE,

and EA, DB when produced meet at C. Prove that the square on DE is equal to the difference of the rectangles $BD.DC$ and $AE.EC$.

7. If a circle can be inscribed in a quadrilateral prove that the sum of two opposite sides is equal to the sum of the other two.

What can you deduce from this theorem in the case in which the quadrilateral is a parallelogram ?

8. ABC is a triangle right-angled at A. On the sides AB, AC squares $ABNM$, $AKLC$ are described external to the triangle ABC. CN, BL cut AB, AC at P, Q respectively. Prove that AP, AQ are, each of them, a mean proportional between BP, QC.

13c

1. Prove that, if all the sides of a convex polygon are produced in order, the sum of the exterior angles is four right angles.

A regular polygon is such that each exterior angle is one-fifth of an interior angle. How many sides has the polygon ?

The angles of a quadrilateral, taken in order, are in the ratio $1 : 3 : 5 : 7$. Prove that two of its sides are parallel.

2. Prove that, in a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

If AD is an altitude of a triangle ABC right-angled at A, prove that the square on AD is equal to the rectangle BD, DC. (This is not to be done by the method of similar triangles.)

3. If, in the same or equal circles, two arcs subtend equal angles at the circumferences, prove that they are equal.

ABC is a triangle inscribed in a circle whose centre is O. D, E, F are the middle points of the arcs BC, CA, AB respectively, and P is the other end of the diameter

through D. Express the angles EOF, BOP in terms of the angles of the triangle ABC, and show that EF is equal to BP.

4. If a straight line touch a circle and, from the point of contact, a chord be drawn, prove that the angles which the chord makes with the tangent are equal to the angles in the alternate segments.

From a point P a tangent PA is drawn to a circle, and a secant PBC cutting the circle at B and C. The bisector AQ of the angle BAC cuts BC at Q. Prove that PQ is equal to PA.

5. Show how to construct a regular hexagon in a circle. Give a proof.

AB and AC are respectively sides of a square and of a regular hexagon inscribed in a circle. Calculate, in degrees, the size of the angle BAC.

6. Tangents are drawn to two equal circles from a point P. The centres of the circles are A, B; and N, the foot of the perpendicular from P upon AB, lies between A and B. Prove that the difference of the squares on the tangents is equal to the rectangle contained by AB and the difference between AN and BN. Hence find the locus of P if the difference of the squares is constant.

7. Prove that the three medians of a triangle are concurrent at a point one-third of the way up each median.

8. Prove that, if two triangles are equi-angular, their corresponding sides are proportional.

Use this theorem to prove the second part of question 2, and also to prove, with the same figure, that $BA^2 = BD.DC$.

14c

1. If two triangles have two angles of the one equal to two angles of the other, each to each, and also one side of the one equal to the corresponding side of the other, prove that the triangles are congruent.

$ABCD$ is a square. Points P, Q are taken in the sides BA, BC respectively such that $BP = BQ$. AQ and CP cut at O. Prove that $AO = OC$.

2. State all the cases you know of two triangles which are not congruent although having three elements of the one equal to the corresponding three elements of the other. (The elements of a triangle are its angles and sides.)

Justify your statements by drawing figures to illustrate them.

3. The equal angles B, C of an isosceles triangle ABC are bisected by lines BY, CX which meet the opposite sides in Y and X. Prove that XY is parallel to BC.

Prove also that BX, XY and YC are all equal.

4. Prove that, in any triangle, the square on the side opposite an acute angle is equal to the sum of the squares on the sides containing the acute angle minus twice the rectangle contained by one of those sides and the projection on it of the other.

Prove also that, in any triangle, the sum of the squares on two sides is equal to twice the square on half the other side plus twice the square on the median to that side.

5. Prove that the angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

A circle APB passes through the centre of a circle AQB; the circles intersect at A and B. Prove that if P lies within the circle AQB and APQ is a straight line, PB is equal to PQ.

6. Prove that, if two circles touch, the centres and the point of contact lie in a straight line.

P is distant 4 inches from the centre of a circle of radius 2 inches, Through P construct two circles of radius $1\frac{1}{2}$ inches to touch the original circle. If the two constructed circles cut again at Q, what fact as to the

position of Q is evident from considerations of symmetry ?

7. Show, without proof, how to draw a circle touching the side BC of a triangle ABC, and touching the sides AB, AC both produced.

If I is the centre of the circle, and AI cuts the circumcircle of ABC at P, prove that PB and PC are equal.

8. If the three sides of one triangle are proportional to the three sides of another triangle, prove that the triangles are equi-angular.

B, C are the points of contact of the tangents from A to a circle. From any point P on the circumference, perpendiculars PL, PM, PN are drawn to BC, CA and AB respectively. Prove that $PL^2 = PM.PN$.

15c

1. If two triangles have two sides of the one equal respectively to two sides of the other, and if the included angles are equal, prove that the triangles are congruent.

A point D is taken in the side BC of an equilateral triangle ABC and an equilateral triangle CDE is described on CD, the vertices A and E being on opposite sides of BC. Prove that $AD = BE$; and if AD produced meets BE at F, prove that the angle BFD is 60°.

2. Prove that triangles of equal area, on the same base and on the same side of it, lie between the same parallels.

If the equal triangles stand on opposite sides of the same base, show that the base, produced if necessary, bisects the straight line joining the vertices.

3. If the square on one side of a triangle is equal to the sum of the squares on the other two sides, prove that the triangle is right-angled.

In a triangle ABC, the perpendicular from A on BC is equal to the perpendicular from C on AB; prove that $AB = BC$.

4. Prove that the straight line joining the middle

points of two sides of a triangle is parallel to the third side and equal to half of it. In a triangle ABC, AD and BE are drawn perpendicular to BC and CA respectively, and intersect at H. P, Q and R are the mid-points of AH, AB and BC respectively. Prove that the angle PQR is a right angle.

5. If the opposite angles of a quadrilateral are supplementary, prove that a circle can be described to pass through its angular points.

Equilateral triangles ABD, ACE are described on the sides AB, AC of a triangle ABC, so that D and E fall outside the triangle; BE, CD intersect at H. Prove that A, E, C, H lie on a circle.

6. Perpendiculars are drawn from the angular points of a triangle to the opposite sides. Prove that they meet in a point.

7. The sides BC, CA, AB of a triangle are 17, 10 and 21 inches in length respectively. Calculate the lengths of the segments into which AB is divided by the foot of the perpendicular dropped upon it from C, and thence find the length of this perpendicular.

8. Prove that the areas of similar triangles are proportional to the squares on their corresponding sides.

In a triangle ABC, right-angled at A, AN is drawn perpendicular to BC. The internal bisector of the angle BAC meets BC at D. Prove that the ratio of the squares on BD and DC is equal to the ratio of BN to NC.

ANSWERS

ARITHMETIC

PAPER 1a. **1.** 300, 906, 543. **2.** (i) $\frac{3}{26}$; (ii) £25 12*s.* **3.** 492 yds.; 126·5 sq. decametres. **4.** 33 sq. yds., 3 sq. ft.; 4 yds. **5.** $\frac{55250}{443} = 124\frac{318}{443}$. **6.** 3 : 2; 14% gain. **7.** ·5 too small; ·0005 ins. **8.** £80,892.

PAPER 2a. **1.** $6\frac{9}{80}$; 27·7. **2.** Saturday; Thursday. **3.** 572·73 dollars. **4.** $7\frac{1}{5}$ acres; 1 min. 14 secs. **5.** £31 8*s.* $8\frac{5}{8}$*d.* **6.** £50 10*s.*; £540. **7.** $31\frac{2}{3}$%; $46\frac{14}{41}$%. **8.** 125.

PAPER 3a. **1.** £295 0*s.* $3\frac{1}{2}$*d.*; average £26 16*s.* 5*d.* **2.** (i) $\frac{134}{2275}$; (ii) 96·5. **3.** £80. **4.** 18 cm. 5 mm. **5.** 5 ac. 4 sq. ch. 264 sq. yds; £11. **6.** 21 ft. 5·6 ins.; £6 3*s.* 1*d.* **7.** £2142; £1428. **8.** £7500; 15%.

PAPER 4a. **1.** $89\frac{13}{1008}$. **2.** £1380. **3.** (i) $\frac{67}{1444}$, (ii) 55·17. **4.** 118·3 and 0·276. **5.** £84 12*s.* 4*d.*; 7*s.* 6*d.* per copy. **6.** 45·93 cents per lb. **7.** £320; £445. **8.** £1250; £3 15*s.* 0*d.* increase.

PAPER 5a. **1.** £165 15*s.* 9*d.* in July; £78 5*s.* 3*d.* **2.** £950. **3.** 40·5%, 47·4%, 12·1%. **4.** 112 yds. **5.** 185 fr. 53 c. per kilogm. **6.** £2 3*s.* per ton. **7.** $76\frac{1}{4}$. **8.** £7 7*s.* $5\frac{1}{4}$*d.*

PAPER 6a. **1.** 90 cms. 4 mms. **2.** 15 miles; 22 miles. **3.** 12*s.* 8*d.* **4.** 4*s.* 1*d.*; £26,888 in excess. **5.** 2978 tons. **6.** S.I. = £319 10*s.*; C.I. = £326 13*s.* 9·3*d.*; difference = £7 3*s.* 9·3*d.* **7.** £352 14*s.* 6*d.* **8.** £3520.

PAPER 7a. **1.** 3. **2.** 125,033 mins.; ·237. **3.** 1,470,000. **4.** 360 yds.; 352 yds. 2 ft. 2 ins. (to nearest inch). **5.** 450. **6.** £74 17*s.* 6*d.*; £38 14*s.* $10\frac{1}{2}$*d.* **7.** £106 19*s.* $1\frac{1}{2}$*d.* **8.** 3*s.* 0·4*d.* in the pound.

PAPER 8a. **1.** 22 hrs. 24 mins. 59 secs. **2.** 1. **3.** $364 = 2 \times 2 \times 7 \times 13$; $598 = 2 \times 13 \times 23$; 8407. **4.** 1,750,126 gallons. **5.** Second is 11·76% dearer than third; £22 3*s.* 6*d.* to nearest penny. **6.** 3·271. **7.** 83; gain of £5 7*s.* 6*d.* **8.** 1·02 c.c.

PAPER 9a. **1.** $\frac{2\sqrt{10}}{7} = \cdot 903$. **2.** $\frac{12}{25}$; $\frac{27}{49}$; $\frac{18}{35}$. $\frac{12}{25} \times \frac{27}{49} = (\frac{18}{35})^2$. **3.** $11\frac{1}{2}$ days. **4.** $\frac{4}{11}$. **5.** ·184. **6.** 6 inches. **7.** £280. **8.** £8000.

PAPER 10a. **1.** ·18813. **2.** $17\frac{27}{64}$ sq. ins.; $1\frac{423}{512}$ cu. ins. **3.** 68·1 litres. **4.** X, Y, and Z pay £7375, £8062 10*s.*, £4062 10*s.* respectively. **5.** Length of side = 34·642 yds.; area of triangle = 519·6 sq. yds.; ratio = ·414. **6.** 38 yds. 1 ft. 2 ins. **7.** $18\frac{18}{19}$ %. **8.** £23 18*s.* 5*d.*; £16 0*s.* 10*d.*

PAPER 11a. **1.** 155,200 gallons. **2.** 53; 102. **3.** £11,428 11*s.* 5*d.* **4.** $\frac{69}{395}$ and $\frac{69}{4049}$; ·017. **5.** $2{,}333\frac{1}{3}$ cu. yds.; $562\frac{1}{2}$ loads. **6.** (i) gain of £22 10*s.*; (ii) loss of £10. **7.** 128 yds. **8.** 73·8 per thousand.

PAPER 12a. **1.** 39,600. **2.** $\frac{1}{5}$; $\frac{23}{4115}$. **3.** (i) 790 in Canada; 3555 in England; 571 tons 7 cwt.; (ii) 6 hrs. 28′ 52″ a.m. **4.** 2·54 cms. to an inch. **5.** 183 sq. yds. **6.** £785 3*s.* $4\frac{1}{2}$*d.* **7.** £1950; $9\frac{1}{2}$%. **8.** 59·04 ft.; 572·32 ft.

PAPER 13a. **1.** 8 mins. 16 secs. **2.** ·5247. **3.** ·621. **4.** $15\frac{3}{8}$%. **5.** £4 0*s.* 4*d.* **6.** 11·8 yards. **7.** $2\frac{1}{4}$%. **8.** £2488 1*s.* and £1,244 0*s.* 6*d.*

PAPER 14a. **1.** $\frac{17}{22}$. **2.** £67 7*s.* 8*d.* **3.** ·731 ins. **4.** £840 per annum. **5.** £120 per annum. **6.** 8 ft.; 374 sq. ft. **7.** 9420·65 francs. **8.** B better by 3*s.* 1*d.*; £103.

PAPER 15a. **1.** $7\frac{1}{5}$. **2.** £74 13*s.* 4*d.* **3.** 7 : 9. **4.** 37·18; 1st has not, since last figure in 3rd decimal place; 2nd cannot, since no square can end in 3. **5.** £196 17*s.* 6*d.*; 4%. **6.** 9 cu. ft. 63 cu. ins.; 10·38 %. **7.** 47·9 m.p.h. **8.** 83.

ALGEBRA

PAPER 1b. **1.** (i) $\frac{3}{2-x}$; (ii) $x = \frac{y+7}{3-2y}$, $y = \frac{2}{7}$ when $x = 3$. **2.** (*a*) $(x-27)(x+3)$, $(5x+7)(3x-5)$; (*b*) $x = 0$ or 5. **3.** 2·9769; ·0231. **4.** (i) $x = \frac{7}{2}$; (ii) $x = \frac{c(a+b)}{a^2+b^2}$, $y = \frac{a^2+b^2}{c(b-a)}$; (iii) $x = -1$ or $-\frac{5}{2}$, $y = 4$ or $\frac{19}{4}$. **5.** 11 : 15. **7.** $x^2 - 6x + 4 = 0$. **8** $\frac{x+3y}{2(2x-y)}$.

PAPER 2b. **1.** $3x^3 + 2x^2 - x + 15$; $a = -4, b = +16$. **2.** (i) $(x+17)(x-3)$; (ii) $(6x-35)(x-3)$; (iii) $(ax-6)(ax+5)$; $a = 2$ or $-\frac{15}{3}$. **3.** 35 feet. **4.** (i) $15\frac{9}{14}$; (ii) 5 or $-\frac{1}{3}$; (iii) 2·57 or −·90. **5.** £3528 and £3390; 1st better by £$\frac{11n+n^2}{2}$ in n years. **6.** (i) 0·02213; (ii) 116·2. **7.** 3·48; $2x^3 - 12x^2 + 21x - 12 = 0$. **8.** $x = 7$; $q = 1$ when $p = 5$.

PAPER 3b. **1.** $2x + 1$; 1·345 or − 3·345. **2.** (i) $\frac{(a-b)[(a+b)x-ab]}{ab(x-a)(x-b)}$; 0; (ii) $9y = 4x^2 + 26x + 148$. **3.** (i) $x = 5{\cdot}4$; (ii) $x = 2\frac{2}{4}\frac{1}{7}$, $y = \frac{4}{4}\frac{}{7}$; (iii) $x = 9$ or 4, $y = 3$ or 8. **4.** £716. **5.** He saves $\frac{a^2}{100}$ %; $a = 20$. **6.** (i) 7700 yards; (ii) 1150 feet. **7.** (i) 0·8974; (ii) $\frac{3}{2}$. **8.** (i) when $x = 6$, $y = -\frac{1}{2}$ and $z = 2$ or $y = 2$ and $z = -\frac{1}{2}$; (ii) 0 correct to 1 decimal place (actually $26 - 15\sqrt{3}$).

PAPER 4b. **1.** (i) $r = \frac{2uv}{u+v}$; (ii) $r - u = \frac{u(v-u)}{u+v}$, $r - v = \frac{v(u-v)}{u+v}$. **2.** $(x-1)(x-2)(3x+2)$; $x = 1$

2 or $-\frac{2}{3}$; $\bar{1}$. **3.** (i) $x = -4$, $y = \frac{7}{2}$; (ii) $\frac{11}{7}$; (iii) $-1{\cdot}309$ or $\cdot509$. **4.** $-\cdot699$ (or $\bar{1}{\cdot}3010$), 1·796, 0·012915. **5.** 3 : 23 or 1st class passengers $13\frac{1}{2}\frac{1}{3}\%$ of 3rd class. **6.** (i) £360; (ii) £405. **7.** $x = 1{\cdot}29$, $y = -\cdot66$ and $x = -1{\cdot}90$, $y = -2{\cdot}61$. **8.** (i) 77 : 118; (ii) $x = \dfrac{1}{14 + 5\sqrt{z}}$, $z = \dfrac{1}{25x^2}(1 - 28x + 196x^2)$.

PAPER 5b. **1.** (i) $\pm 2\frac{2}{5}$; (ii) $\frac{7}{2}$ and $-\frac{3}{2}$; $y = 1 \pm 2\sqrt{x^2+1}$. **2.** (1) $(4x - 3)(2x + 5)$; (2) $(1+ac^2)(1 + bc)(1 - bc)$; $4x - 3$. **3.** 72 miles; 1 hr. 20 mins. **4.** (i) $\dfrac{a - c}{(a + x)(c + x)}$; (ii) 2·64 or $-\cdot24$. **5.** (3) ·3970. **6.** (1) 191 yds.; (2) 11·12 kilogrammes per kilometre. **7.** 11 ascents; 440 feet. **8.** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $\frac{13}{5}$, $\frac{99}{25}$; $125x^2 - 820x + 1287 = 0$.

PAPER 6b. **1.** (1) $\dfrac{2}{(x - 1)(x + 3)}$; (2) $(x - 1)(x + 1)(x - 5)(x + 5)$, $3(x - y)(x + y - 2z)$. **2.** $a = 17$ $b = -12$; $(3x - 4)(4x + 3)$. **3.** (i) $-\frac{25}{7}$; (ii) $x = 4$ $y = 3$ or $x = 32$, $y = -39$. **4.** A 15 goals, B 24 goals. **5.** 4, $\bar{1}{\cdot}6$, ·001; 9·970. **6.** $250\{(1{\cdot}2)^n - 1\}$; £1298 10*s*. **7.** 5·12 or ·88. **8.** (i) $\dfrac{43\sqrt{5} - 1789}{4756}$; (ii) $x = 8\frac{1}{2}$, $y = 9\frac{1}{2}$.

PAPER 7b. **1.** $\pi wh(b^2 - a^2)$; $r = \sqrt{\dfrac{3h(b^2 - a^2)}{4n}}$. **2.** $(2x - y + z)(2x - y - z)$, $(2x - y + z)(2x + y + z)$, $-(2x - y + z)(2x + y - z)$; common factor $2x - y + z$; x. **3.** 12·35 p.m.; $6\frac{2}{3}$ miles past B. **4.** First term 4 or 25; common ratio $1\frac{1}{2}$ or $-\frac{3}{5}$. **5.** (*a*) $x = 6$ or $\frac{1}{2}$, $y = -\frac{3}{2}$ or $\frac{1}{4}$; (*b*) $\frac{3}{5}$; (*c*) 6·29 or $-\cdot79$. **6.** 1·69 or $-0{\cdot}44$. **7.** 202·1. **8.** 3 : 4.

PAPER 8b. **1.** $3x^2-2x+5$; $\frac{3}{x+3}$. **2.** (i) $\frac{8}{3}$; (ii) $x=\frac{ab(3a+2b)}{a^2+b^2}$, $y=\frac{ab(2a-3b)}{a^2+b^2}$; (iii) 3 or -4. **3.** $2xt$ mins; $\frac{1440}{60+t}$ hours; $t=\frac{60}{287}$. **4.** $276\frac{2}{3}$ feet. **5.** Width 1 yard; inner radius 10 yards. **6.** Nov. 5th, by 16 mins. 15 secs.; (i) Dec. 25th; (ii) Oct. 21st and Nov. 18th. **7.** (i) 13; (ii) $W=\frac{5G(3CR-4D)}{2D(G+45)}$; (iii) $W=20{\cdot}08$. **8.** Sum of roots $=-(a+b)$; product $=a^2+ab+b^2$; $\frac{166}{9}$, $-\frac{16}{15}$; $135x^2-2346x-2656=0$.

PAPER 9b. **1.** (1) $\frac{3}{(x-1)(2x-3)(x+2)}$; 0. **2.** 48. **3.** (i) $-1\frac{1}{5}$ or $-2\frac{2}{5}$; (ii) $x=a^2-b^2$, $y=2ab$; (iii) $x=2, y=1$ and $x=12, y=-\frac{17}{3}$. **4.** $2(pq-r^2)\sqrt{pq}$. **5.** (i) $\frac{1}{18}$; (ii) $-{\cdot}75$; (iii) -3; 0·7771. **6.** $a=\frac{1}{4}$, $b=\frac{1}{8}, c=\frac{1}{24}$; expression is $\frac{5x^2+4x-21}{12}$; graphs cut where $x=5{\cdot}26$ or $-1{\cdot}26$, $y=11{\cdot}52$ or $-1{\cdot}52$. **7.** 70 m.p.h. and 80 m.p.h. **8.** (i) $x=66, y=-25$; (ii) $x=3, y=20$ or $x=20, y=3$; (iii) $x=6, y=20$.

PAPER 10b. **1.** 2·601; $\frac{7}{4x-3}$. **2.** (i) $a=11$, $(x-3)(x+2)$; (ii) $(x+y)(7x^2-13xy+7y^2+9x-9y+3)$. **3.** (i) $x=\frac{a^2+b^2}{a-b}$; (ii) $x=2$ or $\frac{11}{9}$; (iii) $x=5\frac{1}{2}$, $y=3$, $z=-\frac{1}{2}$. **4.** 5*d*. each. **5.** 19·83. **6.** 1·147. **7.** (i) 39 or 40 inches; (ii) 10 ins. **8.** $4\sqrt{3}$, 10, 12.

PAPER 11b. **1.** (i) $x=\frac{1}{1-z}$; (ii) x^3+3x^2-4x+2 **2.** $(x+12)(x-2)$, $(3x-4)(x-2)$, $(4x+3)(x+12)$

L.C. Multiple $= (x-2)(x+12)(3x-4)(4x+3)$. **3.** $\frac{8}{x^2-2x-3}$; (i) $2\frac{1}{2}$; (ii) $x = 0{\cdot}6$ and $y = -0{\cdot}3$. **4.** 4 miles; $1\frac{1}{3}$ hours. **5.** First term 24; common difference $2\frac{1}{4}$; 34th term $= 98\frac{1}{4}$; $2078\frac{1}{4}$. **6.** 0·90 or −0·74. **7.** (i) 4·72; (ii) ·001; (iii) $\frac{13}{12}$; 0·531. **8.** $x = 75$ and $y = 80$ or $x = 80$, $y = 75$.

PAPER 12b. **1.** $\frac{x}{(x-1)(x+2)}$; $\frac{7}{18} - \frac{4}{3} + \frac{7}{6} \equiv \frac{2}{9}$. **2.** (i) $(3+4a)(ax-2)$; (ii) $(12x-1)(8x+1)$; (iii) $(x-3y-9)(x+y-1)$. **3.** 4 m.p.h. **4.** (i) 65%; (ii) 53 %; (iii) 44 %; (iv) 36 %. **5.** 67·46 ins. **6.** 11 and 7. **7.** $3x^2 - 14x + 5 = 0$. **8.** 0·031 ohms.

PAPER 13b. **1.** (i) $2(bx-a)$; (ii) $12x^5 - 40x^4 + 77x^3 - 92x^2 + 43x - 6$. **2.** $(x+9)(x-2)$; $(x-3)(x+9)(x-4)$; $(2x-3a)(x+9)$; H.C.F. $= x+9$. **3.** (i) $5\frac{1}{6}$; (ii) 3·88 or −2·13; (iii) $x = 5$, $y = -9$ or $x = 22$, $y = 16{\cdot}5$. **4.** £4400. **5.** 114; 2·4; 2764·8. **6.** $\frac{1}{2}$; 6·5. **7.** 2·071. **8.** $13(7 + 2\sqrt{10})$.

PAPER 14b. **1.** (i) $\frac{Pt+mu}{m}$, $\sqrt{\frac{2Ps+mu^2}{m}}$; (ii) $(x+18)(x-4)$, $(x+2a+3)(x-a+3)$. **2.** (i) $\pm a$; (ii) $x = 2$, $y = -\frac{1}{3}$, $z = \frac{1}{2}$; (iii) 1·17 or 0·16. **3.** 66 eggs; 2*s.* 6*d.* per dozen. **4.** (i) $\frac{9}{2}$; (ii) 0·6053; (iii) 2; 0·217. **5.** $r^{2n} + r^n + 1$; $2\frac{1}{2}$ or $\frac{2}{5}$. **6.** 14 yards at 1*s.* $7\frac{1}{2}$*d.* per yard. **7.** (3, 2), $(-\frac{3}{7}, -\frac{4}{7})$. **8.** (i) 12; (ii) 2·26 ins.

PAPER 15b. **1.** $b = c = -3$; $\frac{2}{x+3}$. **2.** $x^3 + cx^2 + 12x - 4$; (i) $(a-b)^2$; (ii) $(3x-2)(3x^2 + 3xy + 3y^2 - 9x - 12y + 13)$. **3.** (i) −1; (ii) $x = 2{\cdot}5$, $y = -3{\cdot}5$ or $x = 4{\cdot}3$, $y = {\cdot}1$. **4.** $\frac{5\sqrt{2}}{8} = 0{\cdot}884$ ins.

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approx.; 0·391 sq. ins. approx.; [illegible] sq.
5. 0·04719. **6.** 25·99 and 4·99 or $-6{\cdot}1$ and -2.
7. $9\frac{1}{4}$; 0·54 and $-5{\cdot}54$. **8.** (i) $-\frac{2b}{a}$; (ii) $\frac{c}{a}$; (ii)
$\frac{2}{a}\sqrt{b^2-ac}$; $11x^2+5x-3=0$; $33x^2-40x-64=$

GEOMETRY

PAPER 1c. **1.** 9 sides; 15°. **3.** $x\sqrt{3}$ inches.
$AB = 2\frac{1}{3}$ ins. **8.** $10\frac{10}{11}$ sq. ins.

PAPER 2c. **6.** 6·5 ins.

PAPER 3c. **4.** 2·44 inches. **8.** $5\frac{1}{3}$.

PAPER 4c. **3.** $AB = 2{\cdot}4 \times \sqrt{3}$ cms. = 4·157 cms
area of parallelogram = 18 sq. cms.

PAPER 5c. **1.** 45°, 75°, 105°, 135°, 165°, 195°. **3.** 4
sq. ins. **4.** $AB = 1\frac{1}{3}$ ins.; $BC = 2\frac{2}{3}$ ins.

PAPER 6c. **1.** 208, 437, 167 and 351 yards. **4.** B
= 5 ins.; $CD = 1$ in.; $AD = 4{\cdot}899$ ins.; area
$ABC = 14{\cdot}697$ sq. ins.

PAPER 7c. **7.** $BC = 5{\cdot}47$ ins.

PAPER 9c. **1.** Angle $= 180° - \frac{360°}{n}$; $R\hat{A}D = 16$
and $P\hat{R}D = 81°$. **4.** $AP = 2{\cdot}96$ ins.; $AQ = 2{\cdot}81$ in
5. Distance is 6 ins.

PAPER 10c. **4.** 85°.

PAPER 12c. **4.** 8 cms.

PAPER 13c. **1.** 12 sides. **3.** $E\hat{O}F = \hat{B} + \hat{C}$; $B\hat{O}$
$= \hat{B} + \hat{C}$. **5.** 15° or 105°.

PAPER 15c. **7.** If CD is the perpendicular, th
$AD = 6$ ins., $BD = 15$ ins. and CD is 8 ins.

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Author Pratt (A.S.) ( )

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